OPEN CHALLENGE '26 SOLUTIONS

1. ISLAND HOPPING

Assume the contrary, i.e. that an airstrip O can cope with the arrival of six planes from A, B, C, D, E and G.

As the total angle at O is 360°, at least one of the angles at O must be ≤ 60°.

Assume $\alpha = A\hat{O}B \leq 60°$.

As each plane flies to the closest airstrip,

$|AO| < |AB|$ and $|BO| < |AB|$

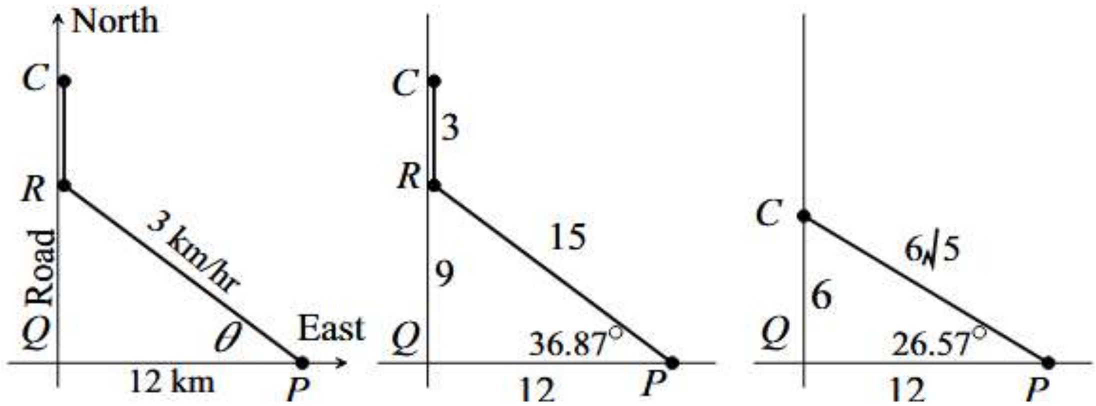
Therefore $A\hat{O}B$ must be the largest angle in the triangle AOB.

Therefore $A\hat{O}B$ must be greater than 1/3 (180°) = 60°.

This is a contradiction.

Therefore no airstrip can cope with the arrival of more than five planes.

2. FLYER'S REST



Referring to the left-hand figure, assume Sue reaches the road at R, so that PRcos θ = 12, i.e. P R = 12/ cos θ, and RQ/12 = tan θ, i.e. RQ = 12 tan θ.

Hence the total time T in hours for Sue to travel from P to C is $\frac{PR}{3} + \frac{RC}{5} = \frac{4}{\cos\theta} + \frac{x - 12\tan\theta}{5}$

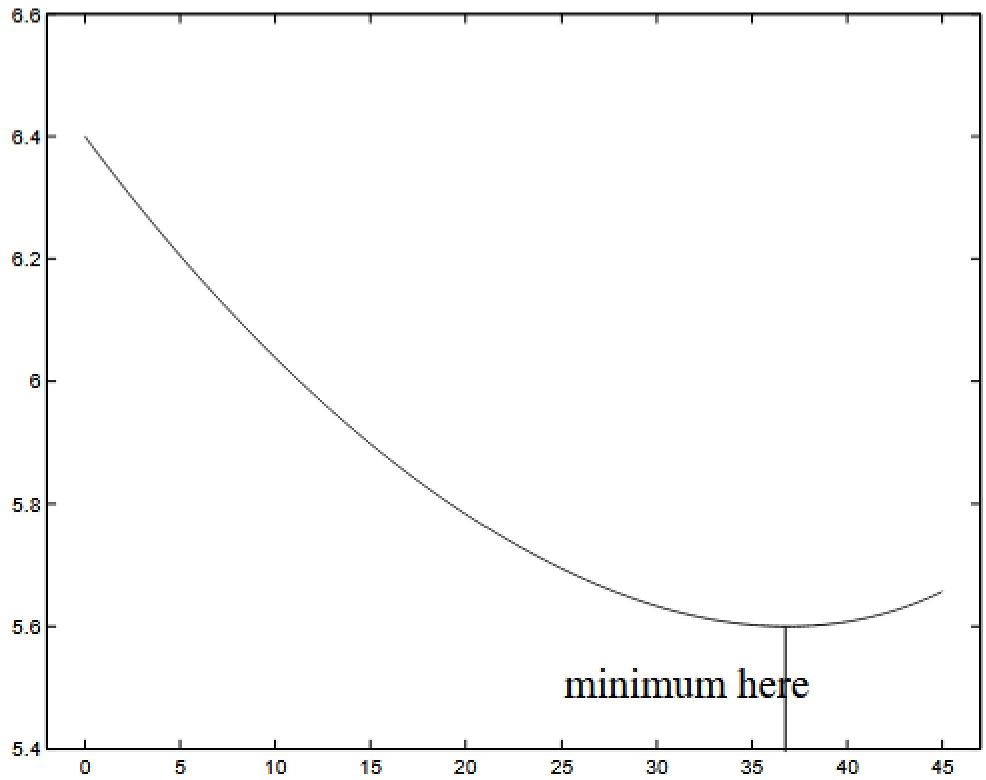
where x = QC. This formula is invalid if R is further north up the road than C. But we do not have to consider this case as the time taken by Sue to go from P to C via R would always be greater than the time she would take if she went directly across the moor from P to C!

Case 1: QC = x = 12 km. Then $T = \frac{4}{\cos\theta} + \frac{12}{5}(1 - \tan\theta)$

Clearly the angle θ must be < 45° in this case for R to be south of C. By calculating values of T, or by plotting a graph, or indeed if you know how to do it by calculus, you find that the smallest value of T occurs when θ is about 37°, and then T = 5.6 hours to 1 decimal place.

See the graph and the middle figure.

(The exact figure is sin θ = 3/5, giving θ = 36.87° to 2 decimal places.)



Case 2: QC = x = 6 km; see the right-hand figure.

Here T $= \frac{4}{\cos\theta} + \frac{12}{5}(1 - \tan\theta) - \frac{6}{5}$

This is just the previous T shifted down by 1.2, so the shape of the graph is the same, with a minimum at about θ = 37°, but it is only valid for 0 < θ < the angle whose tangent is 6/12 = 1/2 ,

i.e. (working to 2 decimal places from now on) 0 < θ < 26.57°.

For larger θ the point R is north of C and so we conclude that the quickest route for Sue to take is with θ = 26.57°

i.e. directly across the moor from P to C.

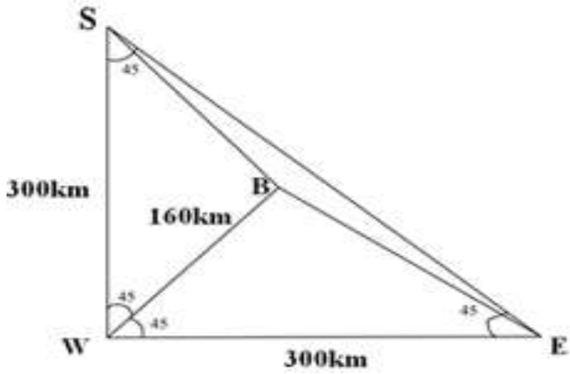
The time taken is 6√5/3 = 4.47 hours.

3. SHARP RETURN

It can be shown that the shortest route is BSWEB (or BEWSB).

There are a number of other routes that would also work.

Time available = 2 hours 39 minutes



Flying time available = 2 hours 39 minutes – 3*25 minutes = 84 minutes.

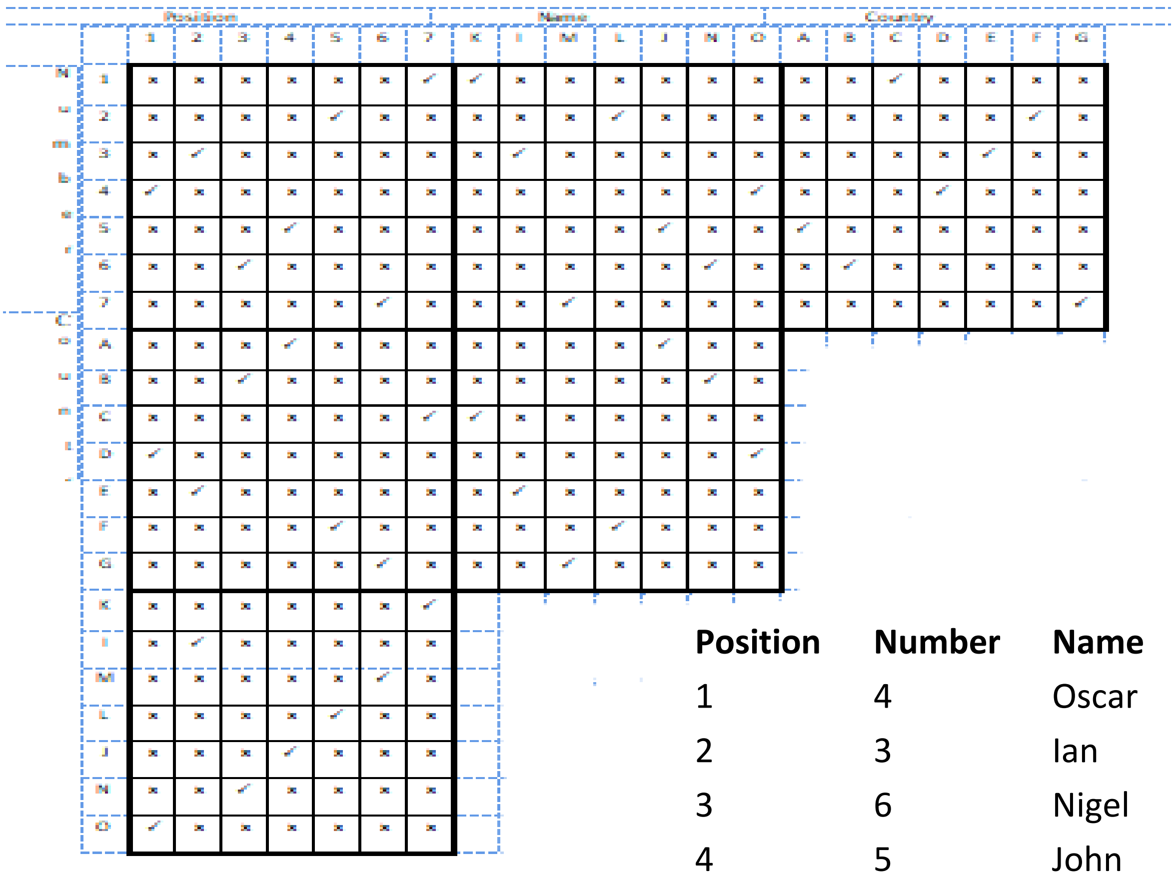
Now $SB^2 = 300^2 + 160^2 - 2*300*160\cos45°$ (cosine rule)
Therefore SB = 218.44 km
Therefore average speed = 218.44/15 km min^{-1}
Time to fly SW = (300 * 15) / 218.44 min = 20.60 min
Time to fly BSWEB = 15 + 20.60 + 20.60 + 15 = 71.2 min
Therefore he has 12.8 min to spare on route BSWEB (or BEWSB)

4.FLYOVER

Take t=0 to be midnight
At time t, θ = ωt (for satellite)
Where ω = 2π/T = 2π/(24 * 60 * 60)
For Liverpool θ = (90 – 53.42) * π/180
Therefore t = 36.58 * π/180 * (24 * 60 * 60)/2π
= 36.58 * 240
= 2 hours 26 minutes 19 seconds (a.m.)
Longitude
Liverpool = 3 * π/180 West
Now $\Phi = \Phi_0 + \omega t$ (moves West as t increases)
Therefore $3 * \pi/180 = \Phi_0 + 36.58(\pi/180)$
Therefore $\Phi_0 = -33.58(\pi/180)$
Hence at time T', satellite is over a point with θ = -π/2 + ωT'
Φ = -33.58(π/180) = ωT' West (if T' <12 hours)
Or -33.58(π/180) +ωT' + π West (if T' >12 hours)
If T' = 7 hours then θ = -90 +360 * 7/24 = 15° South
And Φ = -33.58 +105 = 71.42 = 71°25' West
Which puts it over **Peru**
If T' = 21 hours then θ = -90 +315 = 45° North
And Φ = -33.58 +315 + 180° =101°25' West
Which puts it over **South Dakota in the USA**

5 ORDER, ORDER!

This solution is best achieved using a logic chart.



Position	Number	Name	Country
1	4	Oscar	Denmark
2	3	Ian	Egypt
3	6	Nigel	Belgium
4	5	John	Australia
5	2	Luke	France
6	7	Mark	Germany
7	1	Ken	Canada

6. FLIGHT OF STAIRS

The stack will tip over if the centre of gravity goes past the end of the bottom block.

With two blocks, the top one can overhang by half the length, i.e. 3cm

With 3 blocks, for the combined centre of gravity of the top 2 to be over the end of the bottom block, the middle one overhangs the bottom one by 1.5cm

The process continues, adding another block to the bottom each time, with the overhang being $^1/_3$, then ¼ etc.

$$L = 3 \times \left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \frac{1}{9}\right) = 3 \times \frac{7129}{2520} = \frac{7129}{840} \approx 8.49cm$$