Open Challenge 2023 solutions

1.

2362	Males	Females	Total
Adults	40%	25%	65%
Children	20%	15%	35%
Total	60%	40%	100%

2312	Males	Females	Total
Adults	30%	45%	75%
Children	15%	10%	25%
Total	45%	55%	100%

Let m = number of adult males
and w = number of adult females
In 2312 we know that

w = m + 15%
w + m = 75%
∴ 2m = 60%
m = 30%

Let the population in 2362 be *x* ∴ the population in 2312 is *x* + 30 000
We know that 25%(*x* + 30 000) = 40%*x* ∴ *x* = 50 000
Hence the number of women in 2362 is 25%x50 000 = 12 500
and the number of women in 2312 is 45%x 80 000 = 36 000

2.

Team	Members	Running	Riding
A	**Tahl** **Julah**	**12** **12**	**28** **28**
B	**Ronkol** **Rena**	**16** **10**	**35** **15**
C	**Undina** **Hanak**	**10** **14**	**35** **25**

For Team A as they have identical speeds they will each run half the distance and ride half the distance and finish together.
Hence the time taken will be

21km at 12kmh^{-1} + 21km at 28kmh^{-1} = 1.75 + 0.75 hours
= 2.5 hours

For Team B as Ronkol can run faster than Rena can ride the time taken will be for Rena to ride the 42km

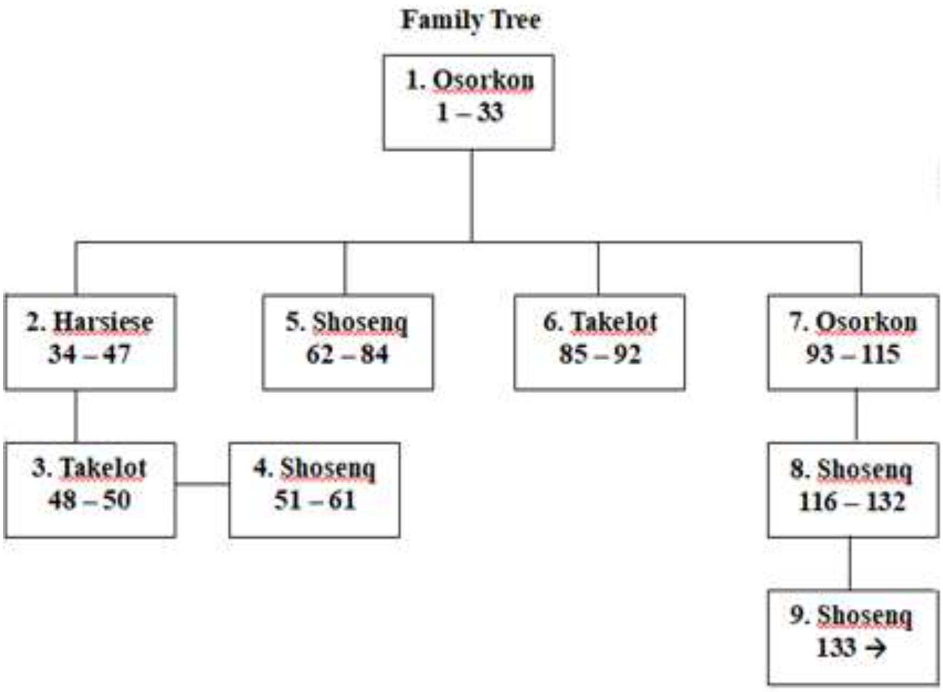
42km ÷ 15kmh^{-1} = 2.8 hours

For Team C the optimum time is when we equate the times for Undina and Hanak.
Hence $\frac{x}{10} + \frac{42-x}{35} = \frac{x}{25} + \frac{42-x}{14}$
This gives a value of 17.5 for *x*
Hence the time taken is $\frac{17.5}{10} + \frac{42-17.5}{35} = 1.75 + 0.7$
This gives a time of 2.45 hours
Thus Team C of Undina and Hanak win the Avaris Cup in a time of 2 hours 27 minutes.

3.

Name of king	Dates
Osorkon	1 – 33
Harsiese (son)	34 – 47
Takelot (son)	48 – 50
Shosenq (brother)	51 – 61
Shosenq II (uncle)	62 – 84
Takelot II (brother)	85 – 92
Osorkon II (brother)	93 – 115
Shosenq III (son)	116 – 132
Shosenq IV (son)	133 – present

For this to work the change of king occurs at midnight on the change of year



4. (a) If Beketaten's first statement is true then she was second (not third); and if her second statement is true then Abar is third (not Beketaten). As there were **no** ties Beketaten's statements prove she was not third.

(b) Dendera's statement that Beketaten was third is false so Abar must be fourth.

(c) This then implies that Beketaten must be second.

Abar's first statement is false so her correct statement gives Dendera as last.

(d) 1st Edjo, 2nd Beketaten, 3rd Chione, 4th Abar and 5th Dendera.

5. A complete cycle consists of a ball being thrown by one hand, caught and held by the other hand before being returned to the first hand. This gives 10 throws and as you are juggling at four throws per second this will take 2.5 seconds per ball (10/4).

As each ball is thrown twice, time between throws for each is 1.25 seconds.

Because you are juggling at four throws per second each hand throws two balls per second or one ball every 0.5 seconds. As your hands are full 70% of the time each ball stays in your hand 0.7x0.5 = 0.35 seconds.

The time each ball is in the air is 1.25 – 0.35 = 0.9 seconds.

The time for each ball to reach its maximum height is thus 0.45 seconds.

At its maximum height the velocity is zero and the acceleration is -9.81ms^{-2}

Using the equation of motion $s = ut - \frac{1}{2}at^2$

$s = 0 - \frac{1}{2} \times -9.81 \times 0.45^2$ = 0.99225m ≈ 1metre.

Thus you must throw the balls approximately 1 metre in the air.

6.

Let the total area of the shield be A.

On the first strike $\frac{1}{2}$ A falls leaving $\frac{1}{2}$ A.

On the second strike $\frac{1}{2}$ of $\frac{1}{2}$ A falls leaving $\frac{1}{4}$ A.

On the third strike $\frac{1}{2}$ of $\frac{1}{4}$ A falls leaving $\frac{1}{8}$ A.

On the fourth strike $\frac{3}{4}$ of $\frac{1}{8}$ A falls leaving $\frac{1}{32}$ A.

Let x be the area of the final piece. Then the total area is $32x$.

Now $\Sigma p = 32x$, where p is a prime number.

The only value of Σp less than 1000 that has a factor of 32 is 160 (i.e. p = 2, 3, 5, ….., 31)

The area that fell to the ground on the first blow was 80 units (7, 19, 23 and 31)

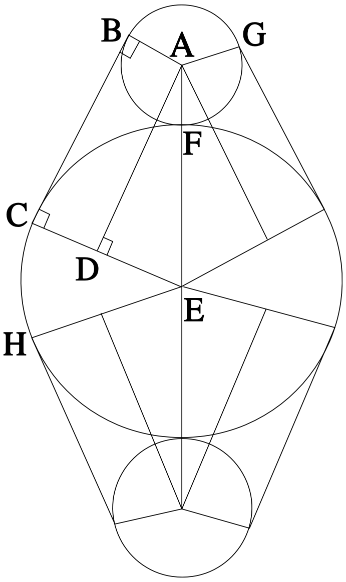
The area that fell to the ground on the second blow was 40 units (11 and 29)

The area that fell to the ground on the third blow was 20 units (3 and 17)

The area that fell to the ground on the fourth blow was 15 units (2 and 13)

This gives the area of the last piece as 5 units.

7.



As the band has a width of 1cm there are 3 options for its length.

The internal length – using radii 4cm and 9cm.

The external length – using radii 5cm and 10cm.

The midway length – using radii 4.5cm and 9.5cm.

Using the first case

$\triangle$ADE is a 5,12,13 right angled triangle

$\therefore$AD = 12cm $\Rightarrow$ BC = 12cm

cos$\angle$AED = 5/13 $\Rightarrow$ $\angle$CEH = 180° - 2cos^{-1}(5/13) = 45.24°

Length of arc HC = $2\pi r\times\theta/360$ $\Rightarrow$HC = 7.106 cm

As BA is parallel to CE

$\angle$BAG = 2×$\angle$AED = 134.76°

Length of arc BG = $2\pi r\times\theta/360$ $\Rightarrow$BG= 9.408cm

Now the length of the band is made up of

4×BC + 2×arcHC + 2×arcBG = 81.03cm = 81cm to nearest cm

Similarly the external length is 87.31cm = 87cm to nearest cm

and the midway length is 84.17cm = 84cm to nearest cm