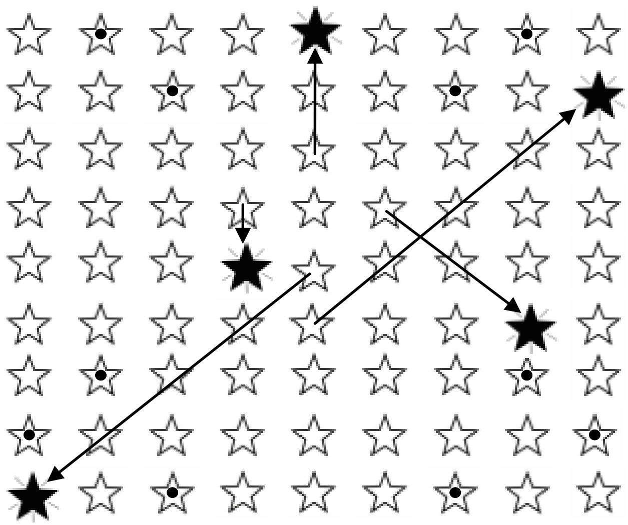
OPEN CHALLENGE '22 Solutions

Girl with a Pearl Earring

Let the middle pearl be worth x pounds.
Then the pearls to the left and right of this each form an A.P. with sum = 2a+(n-1)d
S_L=$\{2(x-100)+(16-1)(-100)\}$=$16x-13600$
S_R=$\{2(x-150)+(16-1)(-150)\}$=$16x-20400$
As the whole necklace is worth £65 000
$16x-13600+x+16x-20400$ = 65000
$\therefore 33x$ = 99000
$\therefore x$ = £ 3000
Thus the value of the large pearl is £3000.

The Starry Night

There are several ways of placing the five planets but it was stated that each planet must obscure five **other** stars in place of those at **present** covered.
Here is one such solution.



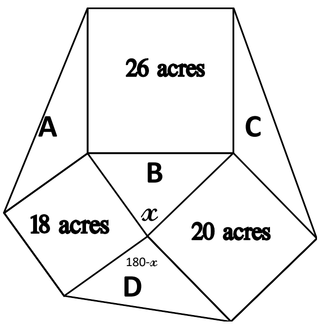
Colour Study: Squares with Concentric Circles

This was based on prime factors.
1 across is abcd = 11111× x (where $x \neq 0$)
The only possible solution is 2×3×41×271 = 66666
Thus the two digits must be 0 and 6.
1 down is 666666=ab^{2}ijkm =2×3^2×7×11×13×37
Thus **a=2** and **b=3** and **c & d= 41 & 271**
1 down divided by 10 across gives j
Anything of the pattern $xxxxxx$ = 11 × $x0x0x$ → **j=11**
2 down and 4 down must contain a factor of 5 to give a 0 on the end.
4 down is a^{2}bcde = abcd×ae = 66666×2e → **e=5**

9 across is now $2^2\times3\times5\times11\times l$ = 6??6?. If **l=101** this gives 66660
Now 6 across is a^2bef = 66060 → **f=367**
Now 2 down $a^2beikj^2 = 4\times3\times5\times91\times121$ = 660660 → **i & k=91=7 & 13**
10 across $ab^2ikm = 2\times9\times91\times m$ = 60606 → **m=37**
3 down $ab^2mnp = 2\times9\times37\times np$ = 600066 → **np=901** → **n & p=17 & 53**
7 across $ab^2gh = 2\times9\times gh$ = 60066 → **gh=3337** → **g & h=47 & 71**
Thus a=2, b=3, c & d=41 & 271, e=5, f=367, g & h=47 & 71, i & k=7 & 13, j=11, l=101, m=37 n & p=17 & 53.

6	6	6	6	6
6	6	0	6	0
6	0	0	6	6
6	6	0	6	6
6	6	6	6	0
6	0	6	0	6

The Harvesters



The area of the four triangles marked A, B, C and D can be shown to be equal to each other.

Triangle B = $\frac{1}{2}\sqrt{18}\sqrt{20}\sin x$ Triangle D = $\frac{1}{2}\sqrt{18}\sqrt{20}\sin(180-x)$

These two equations are equal as $\sin x = \sin(180-x)$.

This process can be repeated for all the triangles.

Using Triangle B

$$\cos x = \frac{18+20-26}{2\sqrt{18}\sqrt{20}} = \frac{1}{\sqrt{10}}$$

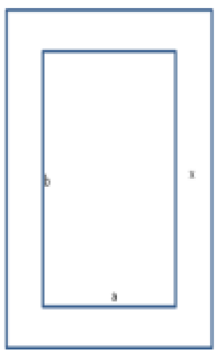
As $\cos^2 x + \sin^2 x = 1$ $\sin^2 x = 1 - \frac{1}{10} = \frac{9}{10}$ Thus $\sin x = \frac{3}{\sqrt{10}}$

Thus area of Triangle B = $\frac{1}{2}\sqrt{18}\sqrt{20}\times\frac{3}{\sqrt{10}} = 9$

Total Area = 4×9 + 26 + 20 + 18 = 100 acres.

Thus he received 100×3×160 = **£48 000**

Mona Lisa



Area of painting =a×b   Total area with frame = 2×a×b

Let new width be aα and new height be bβ. Thus αβ=2.

Any two fractions whose product is 2 will suffice e.g. $\frac{4}{3}\times\frac{3}{2}$ etc.

These measurements are easily obtained.

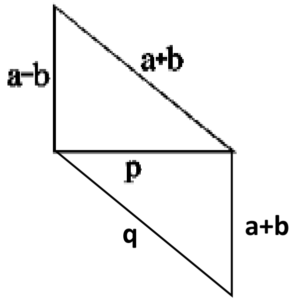
If the width is to be the same, x say, then

$$\begin{aligned} 2ab &= (a+2x)(b+2x) \\ &= ab + 2ax + 2bx + 4x^2 \\ ab &= 2ax + 2bx + 4x^2 \end{aligned}$$

$$4x^2 + 2(a+b)x - ab = 0$$

$$x = \frac{-2(a+b) \pm \sqrt{4(a+b)^2 + 4\times4ab}}{8}$$

$$\therefore x = \frac{-(a+b) + \sqrt{a^2+6ab+b^2}}{4}$$



Using the top right angled triangle

$$p = \sqrt{(a+b)^2 - (a-b)^2}$$
$$p = \sqrt{4ab}$$
$$p = 2\sqrt{ab}$$

Using the bottom right angled triangle

$$q = \sqrt{(a+b)^2 + 4ab}$$
$$q = \sqrt{a^2 + 6ab + b^2}$$

Thus x can be found by taking the length of a+b from q and then dividing the string in half and then half again.

A Sunday Afternoon on the Island of La Grande Jatte

The total distance travelled was 6 x 37.73 = **226.38 miles.**
There were at least 20 riders in the race.

Competitors	Start	Finish	Time Taken	Av. Speed	Position
1 and 2	12.00.00	**14.00.00**	**2 hours**	**113.19 mph**	
3 and 4	12.00.10				
5 and 6	12.00.20	**13.59.37**	**1 hour 59 min 17s**	**113.87 mph**	
7 and 8	12.20.30				
9 and 10	12.00.40				
11 and 12	12.00.50				
13 and 14	12.01.00				
15 and 16	12.01.10				
17 and 18	12.01.20				
19 and 20	12.01.30	**13.59.37**	**1 hour 58 min 7s**	115 mph	FIRST

The greatest average speed was **116.19 mph** achieved by number 1 on his last lap.
Number 1's first five laps took **1h 40min 31s** with an average speed of **112.6mph**.