OPEN CHALLENGE '20
Solutions

The following solutions are very concise. Other methods may also be possible.

1. SQUARE-EYED

Half of all of the squares are odd. As half of 2020 is even, this means that there are an even number of odd squares, which will sum to an even number. The overall sum is therefore even.

2. HIS NAME IN LIGHTS

Letter	Length(cm)	Strips	Product
A	140	4	560
C	110	3	330
E	130	4	520
F	100	3	300
H	120	3	360
I	110	3	330
L	80	2	160
O	140	4	560
P	120	4	480
S	140	5	700
T	80	2	160

Depending on how the letters are formed the lengths, and hence the products, can take different values.
However the six cheapest letters are always L, T, F, C, I and H.
As his first name is Peter he chooses P as his free one.
His name in lights is P. FLITCH as flitch means a side of pork or a steak cut from the side of a fish.

3. I SPY

OPHTHALMOLOGIST encodes to 3 097 659 801 600 000

455 = 5x7x13x(1) which gives E, G, M (plus A's)
This can decode to GEM, MEG, GAME, MAGE, MEGA

114 = 2x3x19x(1) which gives B,C,S (plus A's) or 114 = 6x19x(1) which gives F, S (plus A's)
This can decode to CABS, SCAB, CASABA, ABACAS — No word can be obtained from this

There are three main weaknesses of the code:

1. As can be seen from decoding 114, the code cannot differentiate between anagrams.
2. The non-uniqueness of factorisation (e.g. 12=1x12 or 2x6 or 3x4)
3. As A=1, you cannot tell how many As should be in a word, if any, as seen with 455.

4. THE EYES HAVE IT

The start specs for numbers of passengers, with H, G, B as abbreviations for their eye-colours, are:
R_1 : 6H 4G 2B
R_2 : 4H 4G 4B
(i) requires "4B go from R_2 to R_1" but only 2 transfer, so not possible
P(B = 0 on R_2) = 0
(ii) requires recognition that "(ii) ≡ not (i)"
P(≥1 B on R_2) = 1 – 0 = 1
(iii) requires (2B go from R_1 to R_2) and then (2 out of H or G go to R_2 from R_1)
P(all B on R_2) = $(2/12 \times 1/11) \times {}^{8}C_2/{}^{12}C_2$ = 7/1089 or $(2/12 \times 1/11) \times (8/12 \times 7/11)$ = 7/1089

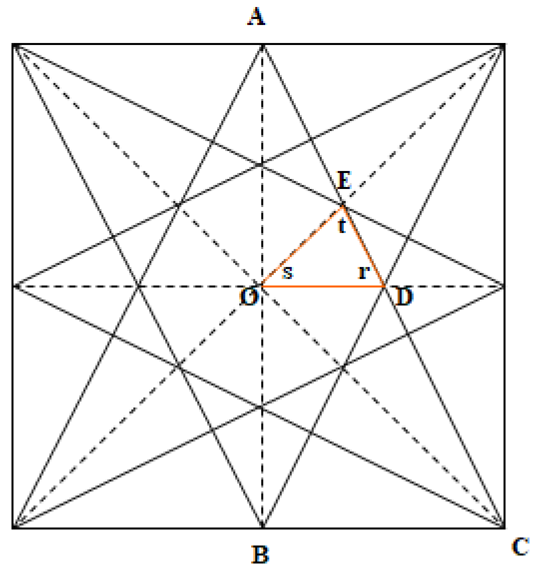
5. COLOUR BLIND

Given that E = 0 then N×D ends in 0 → N or D = 5 and N or D = 2, 4, 8.
N or D ≠ 2 as → G = 1 (impossible) so G = 2 or 4.
As we need values in the 7th and 8th columns G has to be 4. Thus N or D = 5 and N or D = 8.
In the hundreds column R×N = **?**N. This is only possible if N = 5 (→ D = 8) and R is odd (3, 7 or 9).
As we need values in the 7th and 8th columns R ≠ 3. If R = 9 → A = 6 (impossible) thus R = 7.
This gives us 0 ≡ E; 2 ≡ O; 3 ≡ *; 4 ≡ G; 5 ≡ N; 7 ≡ R; 8 ≡ D; 9 ≡ A

6. LOOKALIKES

The electrician labelled the strands of wire at the bottom A, B, C, D, E, F and G. He joined A to B, C to D, and E to F then went to the top of the building. He completed the circuits and detected when he made a correct connection using the circuit tester. He labelled the strands of wire at the top 1 and 2, 3 and 4, and 5 and 6. He knew that G=7 (the only single strand). He then joined 7 to 6, 5 to 4, and 3 to 2.
The electrician then returned to the bottom floor and completed the circuit involving G(7). Suppose this was done with F, thus making F=6. Since E and F and 6 and 5 formed a circuit then E=5. He then completed the circuit with E, say D. Thus D=4 and therefore C=3. By completing the circuit involving C he found 2 equal to A or B (B say) and thus A=1. Thus the electrician was able to identify all the strands of wire having made only one trip to the top of the building.
Instead of grouping the strands into 3 sets of 2 and 1 single it is also possible to use 1 set of 3, 1 set of 2 and 2 singles or 1 set of 3 and 2 sets of 2.

7. AT A GLANCE



The tile has four axes of symmetry. Hence all eight sides of the octagon are equal. Length of each side of the square is 1 foot.
$\therefore BC = \frac{1}{2} foot.\quad OD = \frac{1}{4} foot$ (mid-point theorem ΔABC).
From ΔAOD $\tan r = 2$
By symmetry $s = 45°$
In ΔEOD $t = 135° - r$
By the tangent rule $\tan t = \frac{\tan 135° - \tan r}{1 + \tan 135° \tan r}$

$$= \frac{-1-2}{1-2}$$

$\therefore \tan t = 3$
Hence adjacent angles of the octagon are $2\tan^{-1} 2$ and $2\tan^{-1} 3$
As these are unequal angles the octagon is not regular.
Bert is correct in saying that the sides are equal.
Charlie is correct as the octagon is not regular.

By the sine rule

$$\frac{OE}{\sin r} = \frac{OD}{\sin t}$$

$$OE = OD \times \frac{\sin r}{\sin t} = \frac{1}{4} \times \frac{\frac{2}{\sqrt{5}}}{\frac{3}{\sqrt{10}}}$$

$$OE = \frac{\sqrt{2}}{6}$$

Area of octagon

$$= 8 \times \text{area of } \Delta EOD$$
$$= 8 \times \frac{1}{2} \times OE \times OD \times \sin 45°$$
$$= 4 \times \frac{\sqrt{2}}{6} \times \frac{1}{4} \times \frac{1}{\sqrt{2}}$$
$$= \frac{1}{6}$$

Hence the area of the octagon = $\frac{1}{6}$ area of the square.
Therefore paint is needed in the ratio of red:yellow = 1:5
Therefore the second pot of red paint is not needed and the six pots of paint will cover 600 tiles exactly.