

(INCORPORATING THE LIVERPOOL BRANCH OF THE MA AND THE ATM)

OPEN CHALLENGE '19 SOLUTIONS

1. A STARTER FOR “?”

W H At, I S, 0 Ne, P Lu S, Ni Ne? = Te N = 52 7

2. “DOUBLE” DILEMMA

(a) Using Atomic numbers

$$\frac{(48 + 23 - 17)}{3} - \frac{48}{6} = 10 = \text{Neon}$$

(b) Interpreting the chemical symbols as Roman numerals

$$\frac{(\text{CD} + \text{V} - \text{CL})}{\text{LI}} - \frac{\text{CD}}{\text{C}} = \frac{(400 + 5 - 150)}{51} - \frac{400}{100} = 1 = \text{I} = \text{Iodine}$$

3. ALL THAT GLITTERS

Let weight of gold be G oz and weight of silver be S oz

$$10\text{lb} = 160 \text{ oz}$$

$$G + S = 160 \quad (1)$$

$$\frac{52G}{1000} + \frac{99S}{1000} = 10$$

$$52G + 99S = 10\,000 \quad (2)$$

Solving gives

$$S = 35.74 \text{ oz (2dp)}$$

$$G = 124.26 \text{ oz (2dp)}$$

Therefore the crown was made from a mixture of gold and silver.

4. RINGING THE CHANGES

$$Z \geq \frac{1}{2}T, \quad Z \leq \frac{1}{3}C \quad \text{and} \quad T + Z > 55$$

$$\text{Thus } C \geq 3Z = 2Z + Z \geq T + Z > 55$$

Since Z must be an integer, the smallest value that 3Z can have is 57

Therefore C must be at least 57, not 56.

5. WAR ZONE MEDICINE

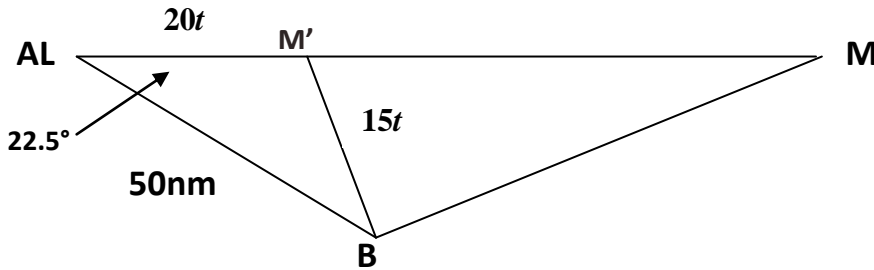
The minimum number of operations required is nine. There are two basic ways with minor variations in the first few moves.

6 fl oz	0	0	0	0	6	0	6	1	1	0	0	6	0	6	1	1	1	1
10 fl oz	0	10	0	5	5	5	5	10	1	10	0	0	6	6	6	10	0	10
15 fl oz	15	5	5	0	0	6	6	6	15	0	10	10	10	10	15	11	11	1

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6. A NAUTICAL ELEMENT

Let AL be the Antoine Lavoisier and B be the Brimstone.
The ships will meet at M (or M') after a time t hours.



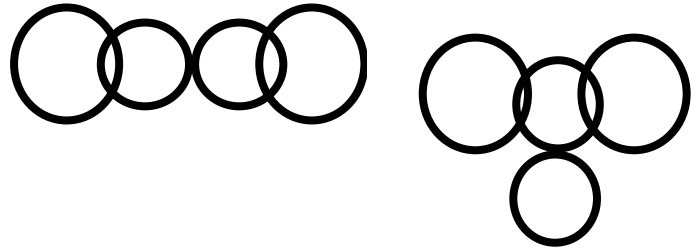
Using the cosine rule

$$(15t)^2 = (20t)^2 + 50^2 - 2 \times 20t \times 50 \cos 22.5^\circ$$

$$t = 1.593 \text{ hours or } 8.965 \text{ hours}$$

7. LUSTROUS LINKS

There are two ways of attaching the platinum link and these will need to be considered separately. When you come to the second one (whichever order you do them in) the arrangements with the platinum link at the ends will be the same as in the first.



(a) 2 gold links not together.
9 links in a row, no restrictions so $9! = 362\,880$
But restriction on gold links so need to remove arrangements where gold links are together
Therefore consider them inseparably joined, giving only 8 links so $8!$
But 2 gold links could be in order AB or BA, therefore $\times 2$ as arrangements, not design
Therefore need to subtract $8! \times 2$
New number of arrangements now $9! - 8! \times 2 = 8! \times 7 = 282\,240$
Platinum link (figure 8) has 2 independent ends, therefore $\times 2$, so now $8! \times 7 \times 2 = 564\,480$
Every link has 2 sides so each one can be attached in 2 ways
So for 9 links, $\{8! \times 7 \times 2\} \times 2^9 = 8! \times 7 \times 2^{10} = 289\,013\,760$
For any given arrangement, simply flipping the chain over through its entire length is not a different arrangement hence divide by 2
Also reversing the ends does not give a different arrangement so divide by 2 again
Answer therefore $8! \times 7 \times 2^{10} / 4 = 8! \times 7 \times 2^8 = 72\,253\,440$

(b) 2 gold links not together and platinum link not at the end as the case where the platinum link is at the end is already counted in (a).
8 non-platinum links in a row, no restrictions so $8! = 40\,320$
7 possible places for platinum link so $8! \times 7 = 282\,240$
But restriction on gold links so need to remove arrangements where gold links are together
Therefore consider them inseparably joined, giving only 7 non-platinum links and 6 positions for platinum link so $7! \times 6$
But 2 gold links could be in order AB or BA, therefore $\times 2$ as arrangements, not design
Therefore need to subtract $7! \times 6 \times 2$
New number of arrangements now $8! \times 7 - 7! \times 6 \times 2 = 7! \times 44$
Platinum link (figure 8) has 2 independent ends, therefore $\times 2$, so now $7! \times 44 \times 2 = 443\,520$
Every link has 2 sides so each one can be attached in 2 ways
So for 9 links, $\{7! \times 44 \times 2\} \times 2^9 = 7! \times 44 \times 2^{10} = 227\,082\,240$
For any given arrangement, simply flipping the chain over through its entire length is not a different arrangement hence divide by 2
Also reversing the ends does not give a different arrangement so divide by 2 again
Answer therefore $7! \times 11 \times 2^{10} = 56\,770\,560$

The jeweller could assemble the chain in a total of 129 024 000 ways.