(INCORPORATING THE LIVERPOOL BRANCH OF THE MA AND THE ATM)

OPEN CHALLENGE '18
SOLUTIONS

1. THE JOURNEY OF ADAM ANT

The simplest solution is when none of the caps cross the straight line between the two bread crumbs. Here Adam travels a distance d<1.6d.
If one or more caps touch or cross this straight line then Adam's maximum deviation will occur when the cap's centre is on the line.
Let d_i be the diameter of one of the caps, then the extra distance travelled is $\frac{\pi}{2}d_i - d_i$.
Thus the total distance travelled by Adam is $d + \sum_1^n \left(\frac{\pi}{2}d_i - d_i\right) = d + \left(\frac{\pi}{2} - 1\right)\sum_1^n d_i$
Now $\sum_1^n d_i < d$ thus Adam travels less than $d + \left(\frac{\pi}{2} - 1\right)d < \frac{\pi}{2}d \approx 1.57d$.
Hence Adam will always be able to travel a distance less than 1.6d.

2. RYAN'S RESTAURANT

(a) A three-course meal can be chosen in $6 \times 5 \times 5$ ways = 150 ways
(b) A two-course meal can be chosen in $6 \times 5 + 5 \times 5$ ways = 55 ways
(c) Following the recommendation a three-course meal can be chosen in
$6 \times 5 \times 5 - 2 \times 2 \times 5$ = 130 ways.
However, as this is only a recommendation it is still possible to choose in 150 ways.
(d) You can choose your meal with no sides, 1 side, 2 different sides or 2 identical sides.

Three-course plus no sides	= 130	(or 150)
Three-course plus 1 side	= 130×5=650	(or 750)
Three-course plus 2 different sides	= 130×10=1300	(or 1500)
Three-course plus 2 identical sides	=130×5=650	(or 750)
Thus a three course meal can be chosen in 2730 ways		(or 3150 ways).

3. APPLE D-CIDER

A reasonable model of the apples is as spheres, while the waste core is a circular cylinder.
By considering the density of the apples it can be shown that there are 27 grade 2 apples per kg.

Apple Type	Grade 1	Grade 2
Diameter (cm)	6	4
Radius (cm)	3	2
Apple Volume (cm^3)	113.09	33.51
Core Volume (cm^3)	4.71	3.14
Useful Volume (cm^3)	108.38	30.37
Cost apple per kg (pence)	80	70
Apples per kg	8	27
Cost per apple (pence)	10	2.59
Useful volume per penny (cm^3)	10.8	11.7

Hence in terms of pie filling the grade 2 apples are marginally more cost effective.
However, there is more waste with them and they take much longer to prepare (more cost in wages).
Thus on balance it would be more prudent to recommend grade 1 apples.

4. CARTER'S CLAIMS

For claim 1 let x be the amount of the Standard cordial used then $\frac{x}{2}$ is the amount of Extra Concentrated used and 1 is the amount of water used. The mixtures in the two containers will have the same strength but different volumes so we scale by a factor α.

This gives $1 + x = \alpha\left(1 + \frac{x}{2}\right)$ hence $\alpha = 2\frac{1+x}{2+x}$

Thus the amount of EC in the mixture is $\alpha\frac{x}{2} = x\frac{1+x}{2+x}$

Now as $x \to \infty$ $\alpha\frac{x}{2} \to x$ However it is unlikely that x is large so for small values of x $\alpha\frac{x}{2} \approx \frac{x}{2}$

If the solution is similar to real life then claim 2 is very nearly true.

For claim 2 the amount of water used with the EC cordial will be $1 + \frac{x}{2}$. If EC is b times the strength of S let the amount of EC cordial used per unit of water be $\frac{x}{b}$. Scaling by $(1 + \frac{x}{2})$ the amount of EC cordial will be $\frac{x}{b}\left(1 + \frac{x}{2}\right) = \frac{x}{2}$ $\rightarrow$ $b = 2 + x$

Thus b is larger than 2, however as x is usually small this is close to claim 1.

Whilst neither claim is strictly true they can be considered close enough.

5. CARLY'S POP-UP CAKE SHOP

Let x be the number of batches of melting moments and y the number of batches of chocolate cookies.
We can now set up a set of inequalities from the given conditions.

$$120x + 240y \leq 22\,500$$
$$96x + 300y \leq 24\,000$$
$$132x + 180y \leq 21\,000$$
$$x \geq 0 \ and \ y \geq 0$$

These are now drawn on a graph and the coordinates which produce the most income are (98.214, 44.643)
As x and y must be integers we need to look at the solutions near to this.
The best solution that fits the given conditions is x=99 and y= 44.
This gives a total income of £1207.80. (Note: we cannot assume that part batches can be made.)

6. THE TOXTETH TORTE

The torte is made up of a cake (carrot and crumble) plus an outer layer of sorbet.
Let x be the length of the cake, y the width and z the height
Then x+2 = length of torte, y+2 = width of torte and z+2 = height of torte and x > z
z_p = 4 as this is 0.2V we can calculate z_c as 6 (as it is 0.3V) both have the same x and y values.
Thus z + 2 = $z_p + z_c + 2z_s$ = 4 + 6 + 2 =12cm
Volume of the carrot and puree layers is 10xy
Volume of torte is 12(x+2)(y+2)
Volume of carrot and puree layers is also 12(x+2)(y+2) – 10xy
Thus 12(x+2)(y+2) – 10xy = 10xy
This simplifies to 3x + 3y + 6 = xy
As we know that x and y must be integers and x > 12 we can use trial and improvement to calculate the values of x and y. This gives x as 18 and y as 4.
Thus the Toxteth Torte is 20cm in length, 6cm wide and 12cm high.