(INCORPORATING THE LIVERPOOL BRANCH OF THE MA AND THE ATM)

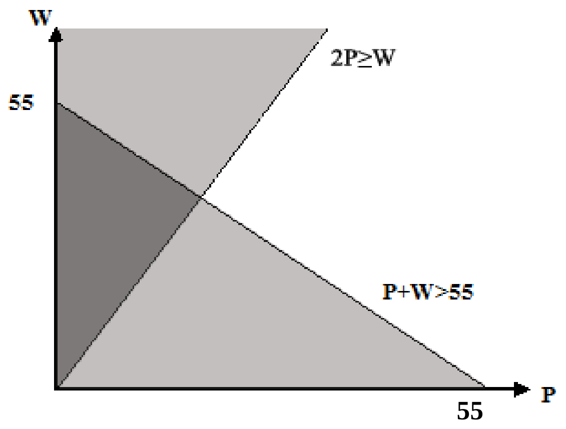
OPEN CHALLENGE '13

SOLUTIONS

1. WHO'S WHO

Ann has 1, Betty has 2, Claire has 3, and Doris has 4 Total 10
This leaves a total of 22 Cavity-Filling Caramels.
This is made up of 4 even numbers or two even and two odd numbers.
It can be shown that it is the second option. (Two 3's and two 8's).
Ann 1 and Glyn 3 Betty 2 and Harry 8 Claire 3 and Frank 3 Doris 4 and Mark 8.
Ann and Glyn Jones, Betty and Harry Robinson, Claire and Frank Smith, Doris and Mark Brown.

2. TAKE YOUR PICK



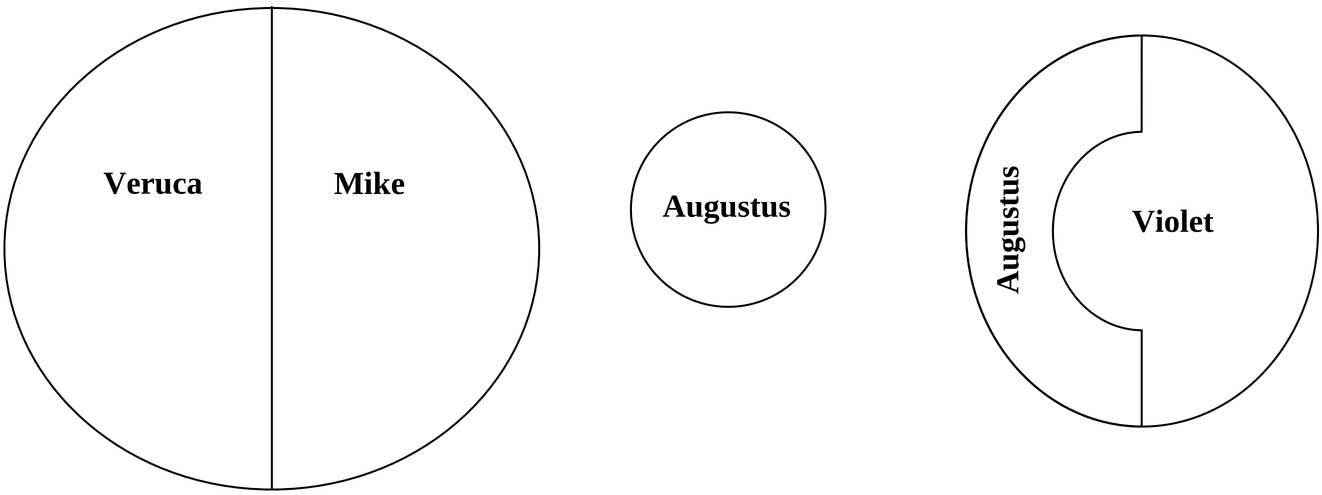
Now 2P≥W, 3P≤M and P+W>55.

Point of Intersection gives
P=55/3 and W=110/3
In order to minimise M we must use the minimum value of P.
This is 19 (as P must be a whole number).
Hence the minimum number of milk chocolates must be 57.
There are 19 plain chocolates and 38 white chocolates giving a total of 114 chocolates in the box.

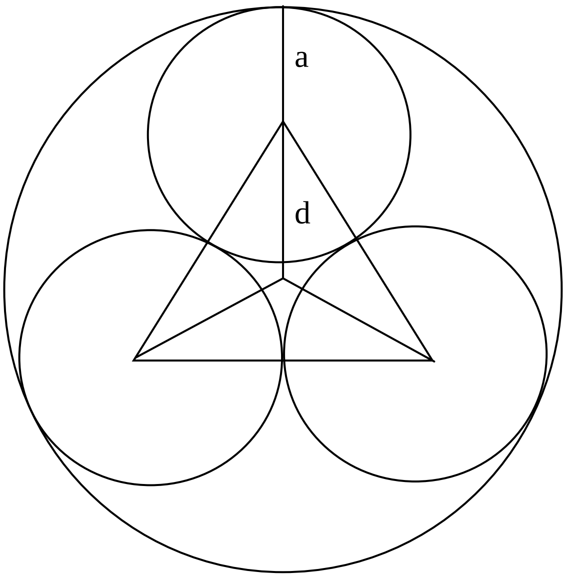
3. A PIECE OF CAKE

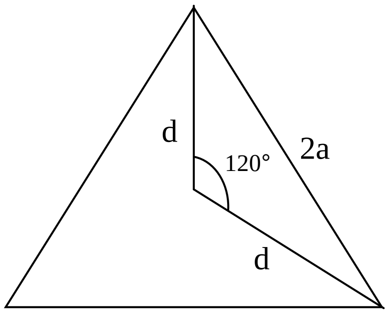
The three diameters form a right angled triangle.
All the cakes are of equal height.
Thus the sum of the areas of the two smaller cakes is exactly equal to the area of the larger cake.

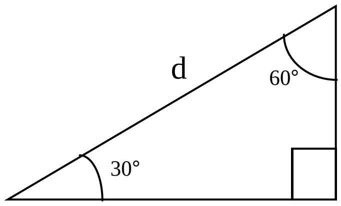
Veruca and Mike each have half of the larger cake (one quarter share).
Place the smaller cake over the other one, cutting as shown, and this gives Violet one piece and Augustus two pieces.



4. SPECIAL CANDY-COATED PENCILS







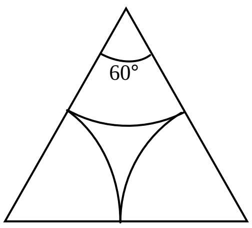
Now $A = a + d$

$$d = \frac{a}{cos30} = \frac{2\sqrt{3}}{3}a$$

$$A = a + \frac{2\sqrt{3}}{3}a$$

$$a = \frac{A}{\left(\frac{3+2\sqrt{3}}{3}\right)}$$

$$a = (2\sqrt{3} - 3)A$$



Central area = Area of triangle – 3xArea of sector

Area of triangle = $\frac{1}{2} \times 2a \times 2a \times sin60 = \sqrt{3}a^2$

Area of sector $= \frac{1}{2} \times a^2 \times \frac{\pi}{3} = \frac{\pi}{6}a^2$

Central Area $= \sqrt{3}a^2 - 3 \times \frac{\pi}{6}a^2 = \left(\sqrt{3} - \frac{\pi}{2}\right)a^2 = \left(\sqrt{3} - \frac{\pi}{2}\right)\left(2\sqrt{3} - 3\right)^2 A^2$

$= 3\left(\sqrt{3} - \frac{\pi}{2}\right)(7 - 4\sqrt{3})A^2$

Thus the volume of the solid chocolate is $3\left(\sqrt{3} - \frac{\pi}{2}\right)(7 - 4\sqrt{3})A^2 h$ or $\left(\sqrt{3} - \frac{\pi}{2}\right)a^2 h$

5. THE WONKA BAR

If the original height of the bar was h units, then the width was $2h$ units and the length was $4h$ units; therefore the volume was $8h^3$ cubed units.

If the new height is k units, then each of the chunks is $4k \times 4k \times k$ units.

With the new shape, if n of the chunks fit across the width, then $2n$ fit along the length.

The new volume of the whole bar is $8nk \times 4nk \times k = 32n^2k^3$ cubed units = $8h^3$.

$\therefore n = \sqrt{\frac{h^3}{4k^3}}$ From this we can work out that n is 4 and the number of chunks is 32.

If the slab had been in the cupboard for t hours, then the steady dwindling of the height means that $k = \left(\frac{120-t}{120}\right) \times h.$

This gives a value of 90 for t.

Hence the slab had been in the cupboard for 90 hours (3days 18hours).

6. THE CHOCOLATE PILE

Let n be the number of sweets at the beginning of the night.

After the first child we have $n-1-\left(\frac{n-1}{3}\right)=\frac{2n-2}{3}$ chocolates remaining.

After the second child we have $\frac{2n-2}{3}-1-\frac{\left(\frac{2n-2}{3}-1\right)}{3}=\frac{4n-10}{9}$ chocolates remaining

After the third child we have $\frac{4n-10}{9}-1-\frac{\left(\frac{4n-10}{9}-1\right)}{3}=\frac{8n-38}{27}$ chocolates remaining

In the morning, after giving one to Red, the remaining number is divisible by 3.

Hence $\frac{\frac{8n-38}{27}-1}{3}\in\mathbb{N}\quad\rightarrow\quad\frac{8n-65}{81}\in\mathbb{N}$

Let $\frac{8n-65}{81}=x$ where $x\in\mathbb{N}$

Thus $8n-65=81x\quad\rightarrow\quad 8n=81x+65$

This implies that x must be odd and $81x+65$ must be divisible by 8.

The smallest value that satisfies these conditions is $x=7$.

This gives a minimum value of 79 for the original number in the pile of chocolates.

Eldest child has 33 chocolates, the middle child has 24 chocolates and the youngest child has 18 chocolates with Red having 4 chocolates.

BONUS

Names	**Confectionery**
Willy Wonka	Cavity-Filling Caramels
Charlie	Candy-Coated Pencils
Veruca	Wonka Bar
Mike	
Violet	
Augustus	
Oompa-Loompa	
Doris	
Charlie’s mother	
Charlie’s father	