$\boldsymbol{Open\ Challenge\ '25}$ **Solutions**

1. FIND THE FUDGE

Let n be the number of bars at the beginning of the night.

After the first child we have $n-1-\left(\frac{n-1}{3}\right)=\frac{2n-2}{3}$ bars remaining.

After the second child we have $\frac{2n-2}{3}-1-\frac{\left(\frac{2n-2}{3}-1\right)}{3}=\frac{4n-10}{9}$ bars remaining.

After the third child we have $\frac{4n-10}{9}-1-\frac{\left(\frac{4n-10}{9}-1\right)}{3}=\frac{8n-38}{27}$ bars remaining.

In the morning, after giving one to Red, the remaining number is divisible by 3.

Hence $\frac{\frac{8n-38}{27}-1}{3}\in\mathbb{N} \quad \to \frac{8n-65}{81}\in\mathbb{N}$ $\quad Let\ x=\frac{8n-65}{81}\ where\ \ x\in\mathbb{N}$

Thus $8n-65=81x \quad \to \quad 8n=81x+65$

This implies that x must be odd and $81x+65$ must be divisible by 8.

The smallest value that satisfies these conditions is $x=7$

This gives a minimum value of 79 for the original number in the pile of bars.

Eldest child has 33 bars, the middle child has 24 bars and

the youngest child has 18 bars with Red having 4 bars.

2. JENNY'S JELLY BEAN FEAST

Due to an error occurring in the question the mathematical solution gives an impossible answer and this should have been mentioned.

The mathematical solution gives:

Let *x be* the total money earned.

$\to \frac{x}{2}=r^2 \quad \frac{x}{3}=m^3$ where r and m are integers

Let n = number of beans eaten → 10n = x

$\to \frac{10n}{2}=r^2\ \ and\ \frac{10n}{3}=m^3$

→ n must be a multiple of 3, say n=3q where q is an integer.

$\rightarrow 15q = r^2 \qquad and \qquad 10q = m^3$

$\rightarrow 2r^2 = 3m^3$

Apart from the trivial solution of r = m = 0, the smallest value

of q is 337 500. This gives a value of n = **1 012 500** jellybeans!!!

To eat **1687** beans (one at a time) per second is not possible!!

3. MELTING MOMENTS

If the original height of the bar was *h* units, then the width was 2*h* units and the length was 4*h* units; therefore, the volume was $8h^3$ cubed units.

If the new height is *k* units, then each of the chunks is $4k \times 4k \times k$ units.

With the new shape, if *n* of the chunks fit across the width, then 2*n* fit along the length.

The new volume of the whole bar is $8nk \times 4nk \times k = 32n^2k^3 \quad = 8h^3$

$$\therefore n = \sqrt{\frac{h^3}{4k^3}}$$

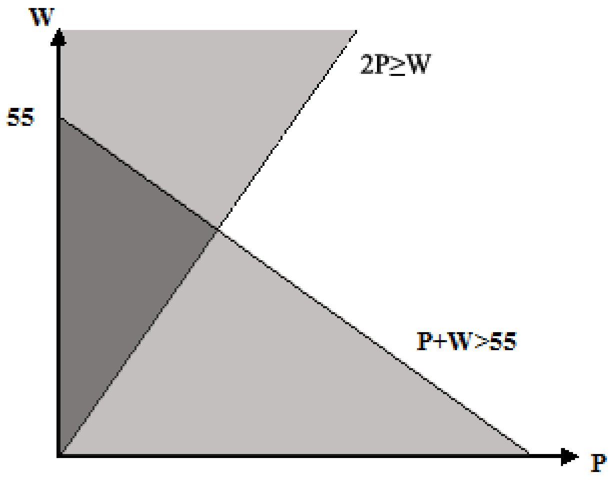
From this we can work out that *n* is 4 and the number of chunks is 32.

If the slab had been in the cupboard for *t* hours, then the steady dwindling of the height means that $k = \left(\frac{120-t}{120}\right) \times h$

This gives a value of 90 for *t*.

Hence the slab had been in the cupboard for 90 hours (3days 18hours).

4. TAKE YOUR PICK



Now 2P≥W, 3P≤M and P+W>55.

Point of Intersection gives

P=55/3 and W=110/3

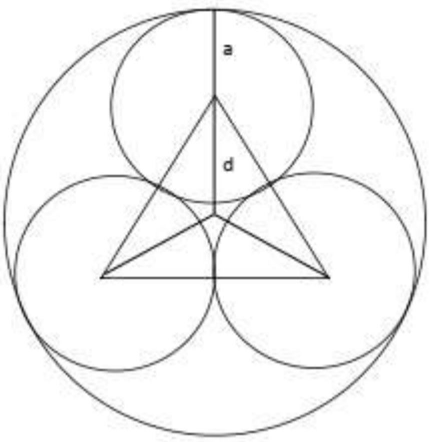
In order to minimise M we must use the minimum value of P.

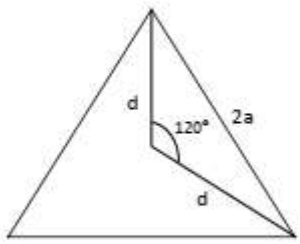
This is 19 (**as P must be a whole number**).

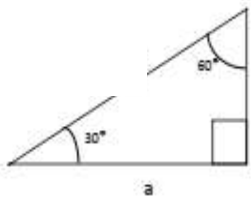
Hence the minimum number of milk chocolates must be 57.

There are 19 plain chocolates and 38 white chocolates giving a total of 114 chocolates in the box.

5. CANDY-COATED PENCILS

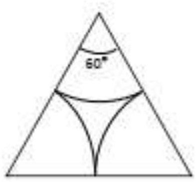






$$A = a + d$$

$$d = \frac{a}{cos30} = \frac{2\sqrt{3}}{3}a$$



$$A = a + \frac{2\sqrt{3}}{3}a \qquad a = (2\sqrt{3} - 3)A$$

Central area = Area of triangle – 3xArea of sector
= Area of triangle – Area of semicircle

Area of triangle = $\frac{1}{2} \times 2a \times 2a \times sin60 = \sqrt{3}a^2$

Area of semicircle $= \frac{1}{2}\pi a^2$

Central Area $= (\sqrt{3} - \frac{\pi}{2})a^2$

Thus, the volume of the solid chocolate is $\left(\sqrt{3} - \frac{\pi}{2}\right) a^2 h$

6. THE GREAT GOB-STOPPER CHALLENGE

The sum of the squares of the first *n* numbers is given by

$$\sum_{r=1}^{r=n} r^2 = \frac{1}{6} n(n+1)(2n+1)$$

Thus, this sum must be a perfect square and so $\sqrt{\frac{1}{6} n(n+1)(2n+1)}$ must be an integer.

Using a spreadsheet, it can be seen that the solutions are *n*= 0, 1 and 24.

As this was a giant square pyramid, we can dismiss the first two solutions and this gives the required solution as a pyramid containing 4900 gobstoppers.

This means that I could win the challenge!