171. What Does It All Mean?

The a.m. = 15, the g.m. = 12.

$$(a-b)^2 = a^2 - 2ab + b^2 \quad = \quad (a+b)^2 - 4ab \geq 0$$

Dividing by 4 and manipulating gives

$$\frac{1}{4}(a+b)^2 \geq ab$$

$$c^2 - d^2 = (c-d)(c+d) \geq 0 \quad \Leftrightarrow \quad c - d \geq 0$$

$$\frac{1}{2}(a+b) \geq \sqrt{ab}$$

172. Tricky!

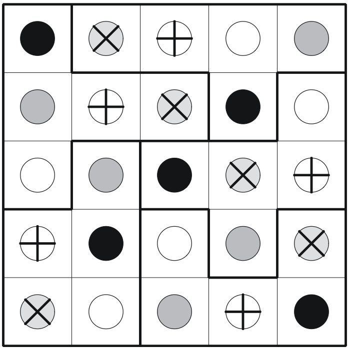
The 3 numbers required are

2^{22}, 3^{33} and 4^{4^4} (**not** 4^{44}).

173. Pentajig

There are several ways of doing this.

One possible solution:



174. Acid Test

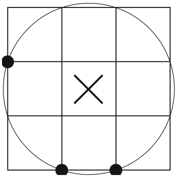
A regular hexagon. The cross-section starts as a triangle, but the other three faces of the cube become involved by halfway down, giving a hexagon.

175. Kurschák's Tile

¾. Just count the equilateral triangles, rhombuses and half rhombuses within the decagon and within the whole square.

151. Central Question

The centre of the circle is in the centre of the diagram:

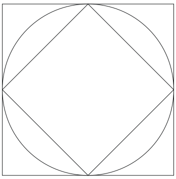


152. The Shape of Things to Come!

L, S, R, U.
The 1st sequence is all letters formed entirely of straight lines.
The 2nd is letters all with curved lines.
The 3rd all have an ‘enclosed’ portion.
The 4th is all letters which have bilateral symmetry.

153. Squares Squeezing a Circle

The answer is ½. This can be checked by rotating the inner square through 45°:



154. The Anglers’ Arms

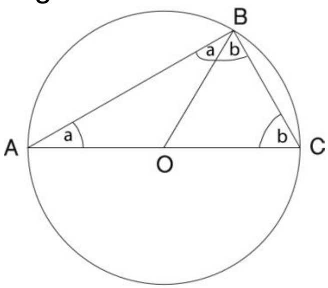
NET < TIN < ROD < FLY
This shows understanding of the fact that the size of an angle refers to its turn and not to the lengths of its arms.

155. Walk Round a Triangle

The walk rotates you through 360°, since you are now facing the same way as at the start.
The sum of all the angles is 180°+ 180° + 180° = 540°.

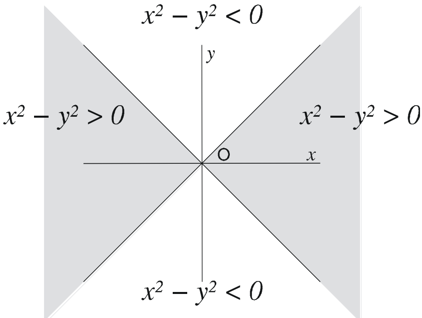
156. Angle in a Semicircle

Add another radius from O to B. This gives two isosceles triangles.



2a + 2b = 2(a + b) = 180°, so a + b = 90°, as required.

157. $x^2 - y^2$



158. Clueless!

There are 65 black squares and $15^2 = 225$ squares altogether,
so the fraction is $\frac{13}{45}$.

159. Broken Chords

There are two choices of similar triangles: add AC & DB, or AD & BC. Either equation will do in either case.

160. Sacrobosco

$$s = \frac{(u+v)t}{2}$$

161. Golden Triangles

Let τ be the length of a side of any of the larger triangles, and 1 the length of a side of any of the smaller ones. Choose one chord breaking into two pieces of length τ and another intersecting it, breaking at that point into pieces of length 1 and $1 + \tau$. Then apply the theorem proved in 159.

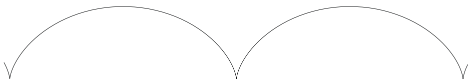
162. Number Maze

⇒	**8**	–2	÷3	+5	**8**	⇐
	+4	×2	+4	×2	+6	
	×3	+4	÷3	×10	+4	
	-2	÷4	+10	+6	÷4	
	+3	×3	–2	÷4	–1	

8	**8**	**8**	**8**	**8**

163. A Move in the Right Direction?

Up and to the right. The path of the marked point is a *cycloid*:



164. Identity Parade

$y = x^2$	$y = x(x^2 - 1)$	$y = \frac{1}{x^2}$
$y = x$	$y = x^3$	$y = \frac{1}{x}$

165. On the Line?

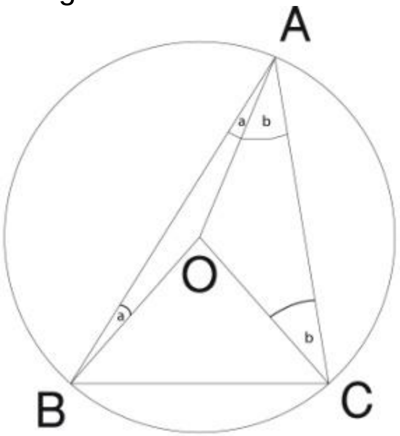
The missing line is AD.
As A and B are diameters, the angles ADB and ADC are both right angles, so BDC is a straight line. So D lies on line BC.

166. Order?

$$4^{2^3} < 4^{3^2} < 3^{2^4} = 3^{4^2} < 2^{4^3} < 2^{3^4}$$

167. Angle Wrangle

Extend a radius through the centre to give two isosceles triangles.



The angle at O is thus

$360° - (180° - 2a) - (180° - 2b)$
$= 360° - 180° + 2a - 180° + 2b$
$= 2a + 2b = 2(a + b)$

This is twice the angle at A.

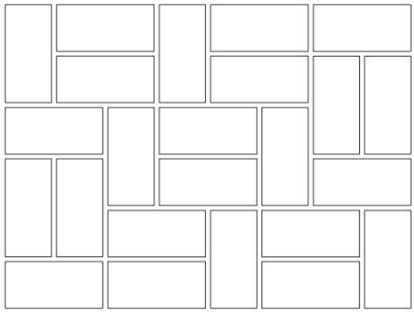
NB The same is true when A is close enough to C for the line AB to intersect the line OC, but more complicated algebraic manipulation is required.

168. You Can't Be Serious!

You should get 4 every time!

169. Faultless

There is more than one solution. The nicest is probably:



Notice that each of the five internal horizontal lines and each of the seven internal vertical lines is bridged by exactly two dominoes.

170. Penny Farthing

$$(r + s)^2 - (r - s)^2$$
$= (r + s + r - s)(r + s - r + s)$
$= (2r)(2s)$

Position the triangle so that the hypotenuse joins the two centres of the circles.