## 6. IT'S IN THE STARS

In a Galaxy far, far away travel between stars is possible along wormholes.
In the constellation Novem there are nine stars, Kochab, Lesath, Maia, Nunki, Procyon, Rastaban, Spica, Tejat and Unukalhai. The map shows the relative positions and the distance in light years between each pair of adjacent stars.
Interstellar travel can only be along these 12 wormholes.
Unfortunately, the names of the stars have been left off the map.

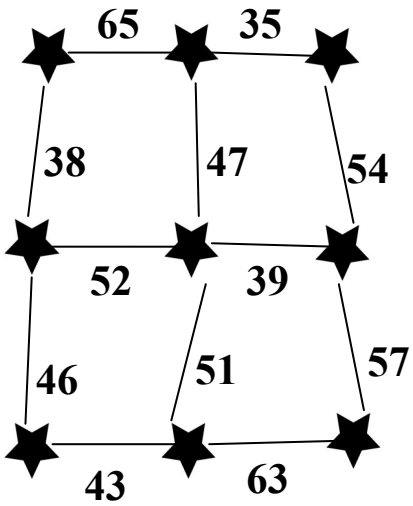


Diagram is **not** to scale.

Luckily we have the following information about the distances, which may be 'multi-hop', between some of the stars.
The two stars that are furthest apart are Spica and Kochab.
The distance from Spica to Unukalhai is the same as the distance from Tejat to Maia.
The distance from Procyon to Nunki is 1 light-year more than the distance from Procyon to Maia.
The distance from Unukalhai to Rastaban is twice the distance from Unukalhai to Procyon.
(Each distance means the shortest distance by a suitable choice of wormholes).
Can you place the stars in their correct positions?

The competition is promoted by Liverpool Mathematical Society (LivMS) www.livmathssoc.org.uk
The competition is sponsored by the Worshipful Company of Actuaries Charitable Trust
The MA is a Registered Charity (No. 313281). Drawings by P. H. Ackerley.

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# Open Challenge '24
## For Year 13 or below

Rules

It should be attempted at home during February half term.
Your entry must be your own work.
For individual entries only. You should attempt all questions.
Entries without any working out at all or written on this sheet will not be marked.
It is possible to win a prize even if you have not completed all of the questions, so hand in your entry even if it is not quite finished.
**You must print your name, date of birth and school in neat, legible writing on the front sheet.**
Pupils under 15 years of age should only attempt this in exceptional circumstances.

Either you or your maths teacher needs to **return your entry by 8 March** to this address:

Open Challenge '24 Entries
Mrs A. Carter
Danes Court
Mudhouse Lane
Burton
Neston
CH64 5TS

An evening of online activities will be held in early May during which there will be a virtual prizegiving.
Prizes and certificates **will be posted** to schools and colleges.
Solutions will be posted on www.livmathssoc.org.uk shortly afterwards.
We hope that you enjoy the questions.

## 1. EXILED!

The Scorpians exile their criminals to a penal colony in a remote star system. Its Sun is called Globe 1, and the surrounding planets are Globe 2 to Globe 100.
Convicts begin their sentence by being transported to a specific Globe determined by their courts.
Each year a convict is moved to a fresh globe according to the following rules:

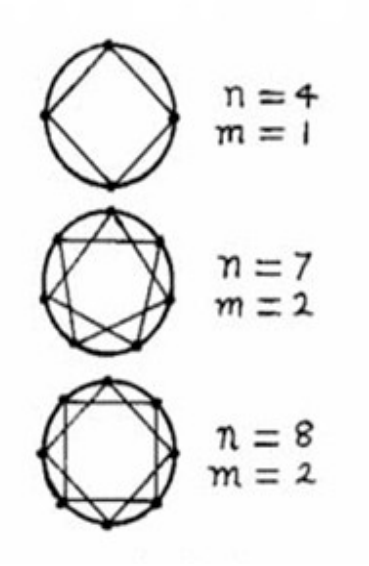
on an even numbered Globe halve the Globe number

on an odd numbered Globe multiply the Globe number by three and add 1.

The sentence ends when the convict is sent to Globe 1, or achieves a globe number greater than 100.
Which starting Globes provide the shortest sentence for the convicts?
Which starting Globe provides the longest sentence for the convicts?

## 2. SEEING STARS

Here is a good way to draw stars. Start with a circle and mark a number of points (say *n* points) equally spaced around the circumference.
Now join each point in turn to the one *m* points further on, as shown.
Notice that the third star consists of two like the first star, but the second star cannot be split into smaller ones.



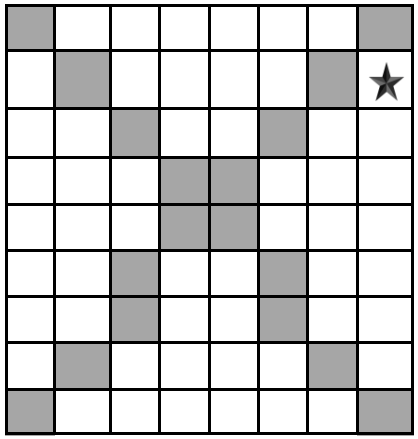

How many different stars of the unsplittable type can be drawn with *n* = 7, 8, 9, 10 and 15?

Make a table of your results.

Find the property which *n* and *m* must have for the corresponding star to split up into two or more smaller stars.

Use this result to decide how many unsplittable stars can be drawn with *n* = 42.

## 3. STAR SCATTER



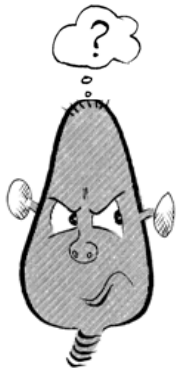
Is it possible to place eight stars in the diagram so that no star is in line with another star horizontally, vertically or diagonally?
One star is already placed, and that must not be moved, so there are only seven more stars to be placed.
However, no star can be placed on one of the shaded squares.

Without the first star being so placed can you find a different arrangement?

## 4. FROM THE EARTH TO THE MOON

I was watching the Moon set the other night, and the time from when it first touched the horizon to the time it completely disappeared was about 56 minutes.
Assuming the Moon's diameter is 3500 km and that it orbits once every 28 days, and ignoring both the effect of the atmosphere and the Earth's rotation, roughly how far away is the Moon?

## 5. HOW FAR TO THE STAR?




Four stars in the constellation OC24 are found, remarkably, to lie in a plane in space, while no three of the four lie in a straight line.
Five of the six distances between these four stars are 3,4,5,8,8 parsecs.
Find all possible values for the sixth distance, to 2 d.p., explaining your reasoning carefully.