## 6. LOOKALIKES

A fault is discovered in an electrical cable containing seven identical strands of insulated wire which runs from the top to the bottom of the Liver Building. The electrician discovers that there is no visible difference between the wires as they are not colour coded and so has to identify which ends at the top connect to which at the bottom. He has a circuit tester consisting of a battery and a bulb, which lights when a circuit is completed between the terminals, but no other equipment or assistance apart from some insulating tape and a marker. Unfortunately as a result of the fault the lifts are out of action but after a little thought he is able to identify all the wires, having made only one trip to the top of the building. How did he do it?

7. AT A GLANCE
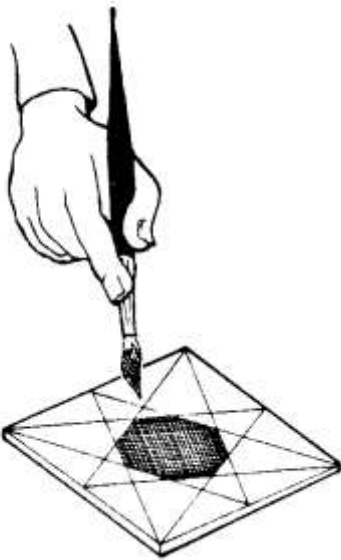
A pattern is produced on a square floor tile by drawing a pair of lines from each corner to the midpoints of both of the sides not containing that corner. Bert is going to paint the octagon on each tile red and Charlie is going to paint the remainder of the tile yellow. "This is a regular octagon," says Bert "all the sides are equal". "You're wrong," retorts Charlie "look more carefully". Is Charlie correct or can they both be right?
Bert has two pots of red paint and Charlie has 5 pots of yellow paint, and each pot will cover 100 square feet of tile surface. Each tile is a foot square. How many whole tiles can they paint?

Open
Challenge '20
For Year 13 or below

Rules

1) It should be attempted at home during February half term.
2) Your entry must be your own work.
3) For individual entries only. You should attempt all questions.
4) Entries without any working out at all or written on this sheet will not be marked.
5) It is possible to win a prize even if you have not completed all of the questions, so hand in your entry even if it is not quite finished.
6) **You must print your name, date of birth and school in neat, legible writing on the front sheet.**
7) Pupils under 15 years of age should only attempt this in exceptional circumstances.

Either you or your maths teacher needs to return your entry by 6 March to this address:

Open Challenge '20 Entries,
Mrs A. Carter,
Danes Court,
Mudhouse Lane,
Burton,
Neston.
CH64 5TS.

All of the prizes and certificates will be awarded at an evening of mathematical recreation at the University of Liverpool on 29 April. Solutions will be posted on www.maths.liv.ac.uk/lms.html shortly afterwards. We hope that you enjoy the questions.

1. SQUARE-EYED

The sum of a series of consecutive square numbers looks like this: $1 + 4 + 9 + 16 + …$
Without calculating, is the sum of the first 2020 terms in this series odd or even? How do you know?

2. HIS NAME IN LIGHTS

The Lumen Illuminated Sign Company is closing down and have a special offer on all remaining stock – 'Buy six different letters and have a seventh letter free'. The only letters left are

ACEFHILOPST

all formed using vertical and horizontal strips only, each strip having width 10cm and the length being one of 20, 30, 40 or 50cm.

The maximum width and height of each letter are 40 and 50cm respectively. The cost of each letter depends on the product of the number of strips and the total length.

Peter is setting up his own butcher-cum-fishmonger business and decides to take advantage of the offer and to have his six letter surname and one initial in lights above the shop. He is pleased to discover he need only pay for the six cheapest letters. Which letter does he choose to have free, and what is his appropriate but unusual surname?

3. I SPY

A simple code is devised, the first step of which is to replace each letter of a word by a number, and the process is completed by forming the product of the numbers used. The initial replacement is done with 1 instead of A, 2 for B and so on. (Thus OPEN encodes as $15 \times 16 \times 5 \times 14 = 16800$, and CHALLENGE becomes 8467200).
Encode OPHTHALMOLOGIST.
Decode 455 and 114.
What are the weaknesses of this cipher?

4. THE EYES HAVE IT

Two distinct bus services, labelled route R1 and route R2, operate through a sparsely populated area. Their routes intersect at a particular isolated crossroads location which acts as an interchange point where timings of the services on the two routes are such that passengers can readily transfer between them. Being "in the middle of nowhere", no additional new passengers join or leave either.

One day the bus operating route R1 en route to the interchange is carrying 12 passengers – 6 Hazel-eyed, 4 Green-eyed and 2 Blue-eyed. The intersecting bus operating on route R2 is also carrying 12 passengers, in this case 4 of each of the three eye colours indicated.

At the interchange there are 2 'random' passengers who transfer from the R1 bus to the R2 bus and, at the same time, 2 from the R2 bus to the R1 bus. Calculate the probabilities of the following outcomes following the interchange.

(i) none of the blue-eyed passengers is on the bus on route R2.

(ii) at least one of all the blue-eyed passengers are on the bus on route R2,

(iii) all of the blue-eyed passengers are on the bus on route R2.

5. COLOUR BLIND

Question: What do you get when you cross GREEN with RED?

Answer: **ORANGE !

$$\begin{array}{r} \text{GREEN} \\ \text{RED} \times \\ \hline **\text{ORANGE} \end{array}$$

Each letter and the * stands for a number from the set {0, 2, 3, 4, 5, 7, 8, 9} and no two letters stand for the same number. Given that E is zero, match the numbers to the letters and the *.