(INCORPORATING THE LIVERPOOL BRANCH OF THE MA AND THE ATM)

OPEN CHALLENGE '17

SOLUTIONS

1. TERRY'S TARGET

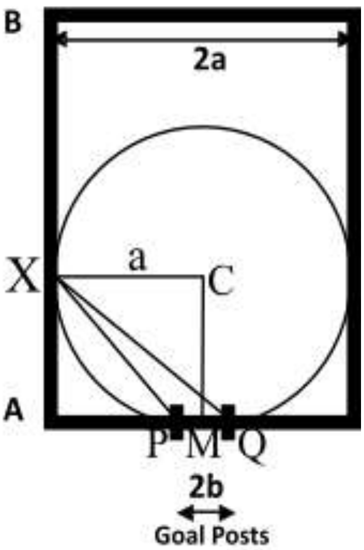
E(X) =0.4×N + 0.3(N_L+N_R) where N is the number aimed at and N_L and N_R are the numbers to the left and right of N. This gives

N	1	2	3	4	5	6	7	8	9	10
E(X)	11.8	10.4	12.0	10.9	11.6	9.3	13.3	11.3	11.4	10.3
N	11	12	13	14	15	16	17	18	19	20
E(X)	11.0	9.0	8.2	11.6	9.6	10.9	8.3	8.7	10.6	9.8

Thus Terry should aim for 7 as this gives the greatest expected value.

After practice 70% is the original E(X), 20% is double E(X) and 10% is triple E(X).
This now gives 1.4 × E(X) and so Terry should not revise his strategy.

2. KEVIN'S KICK



The optimum position is given by a circle which passes through the goal posts P and Q, and touches the line AB.
Let C be the centre of the circle and M be the midpoint between P and Q.
In ΔMCQ QC = a and MQ = b

$CM^2 = a^2 – b^2$

Thus CM = $\sqrt{a^2 - b^2}$

Thus Kevin should take his kick from X, $\sqrt{a^2 - b^2}$ from A.

3. 2 DIE 4

$E_4(X) = \frac{1}{4}(1 + 2 + 3 + 4)=2.5$

$E_6(X) = \frac{1}{6}(1 + 2 + 3 + 4 + 5 + 6)=3.5$

$E_8(X) = \frac{1}{8}(1 + 2 + 3 + 4 + 5 + 6 + 7 + 8)=4.5$

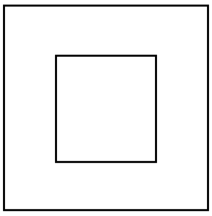
$E_{12}(X) = \frac{1}{12}(1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12)=6.5$

Multiplying these by the number of dice gives expected scores of 15, 14, 13.5 and 13 respectively. Hence you should choose 4-faced dice.

N	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
$E_4(X)$					.0014	.011	.052	.176	.469	1.05	2.03	3.48	5.35	7.47	9.60	11.5	13.0	13.97	14.6
$E_6(X)$			.003	.019	.065	.173	.389	.778	1.40	2.28	3.44	4.84	6.42	8.04	9.58	10.9	12.1	12.9	
$E_8(X)$		.006	.029	.088	.205	.410	.738	1.23	1.93	2.84	3.91	5.13	6.45	7.79	9.11	10.3	11.3	12.1	
$E_{12}(X)$	.014	.056	.139	.278	.486	.778	1.17	1.67	2.29	3.06	3.97	5.06	6.13	7.17	8.17	9.11	9.99	10.8	

Using the above values $2 \leq N \leq 12$ use the 12-faced dice; $13 \leq N \leq 14$ use the 8-faced dice;
N=15 use the 6 faced dice and $16 \leq N \leq 23$ use the 4-faced dice.

4. FAIR GAME?



Place a 2.5cm square at the centre of each individual 5cm square. The centre of the 2p coin must lie inside the smaller square if it is to lie fully inside the larger square.

Hence P(coin does not touch a side) = $\frac{2.5^2}{5^2} = \frac{1}{4}$ = 0.25

E(x) = (0×0.75) + {(90×2) + (4×5) + (3×10) + (2×20) + (1×50)}×0.0025
= 0.8p

Hence the game makes a profit of 1.2p and the fraction given out as prizes is $\frac{0.8}{2} = \frac{2}{5}$.

For all money to be returned E(x) = 2p thus 2 = $\sum x \times 0.0025$
Thus $\sum$ x = 800. This can be achieved in several ways.
For example 80 blank, ten 50p, five 20p, three 10p and two 5p squares.

5. RUNAWAY RESULTS

This is done by logically going through each team and comparing their results with each other team.
This will give us the unique set of results shown here.

RESULTS							
AvE	**1 - 0**	**CvH**	**2 - 1**	**AvG**	**3 - 1**	**CvA**	**1 - 1**
BvJ	**3 - 0**	**DvI**	**4 - 0**	**BvH**	**1 - 1**	**FvD**	**0 - 2**
GvC	**1 - 1**	**EvB**	**1 - 1**	**DvC**	**0 - 2**	**GvJ**	**2 - 1**
HvD	**0 - 0**	**FvG**	**6 - 2**	**EvI**	**2 - 0**	**HvE**	**0 - 1**
IvF	**2 - 1**	**JvA**	**1 - 3**	**JvF**	**2 - 3**	**IvB**	**1 - 3**

6. SOCIAL SQUASH

For n people to play each other the first person will play (n-1) games with everyone else. Thus the number of games will be n(n-1)÷2 (as each game involves 2 people). As each player plays each other twice the total number of games is n(n-1)

Total number of games = 11×12
= 132

Total cost of hire = £5×132
= £660

As there are 6 games per week the number of weeks required is
$\frac{132}{6} = 22$ weeks.

Let the number of friends be n, where n>12.
The total number of games is = n(n-1)
And the cost of hire will be = £5n(n-1)

When n is even there will be $\frac{n}{2}$ games per week and all matches will take place in n(n-1)÷ $\frac{n}{2}$ =2(n-1) weeks.

When n is odd there will be $\frac{n-1}{2}$ games per week and all matches will take place in n(n-1)÷ $\frac{(n-1)}{2}$ = 2n weeks.