(INCORPORATING THE LIVERPOOL BRANCH OF THE MA AND THE ATM)

OPEN CHALLENGE '16
SOLUTIONS

1. THE LAST BANG

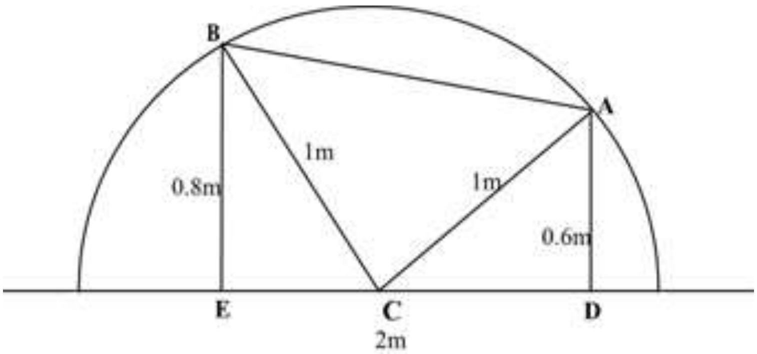
As Time = Distance ÷ Speed
We have

Fragment	Time Since Explosion
1	4.0×10^9 s
2	4.0×10^9 s
3	4.0072×10^9 s

This shows that Fragments 1 and 2 were probably the result of one explosion whereas Fragment 3 came from an earlier explosion.

4 000 000 000s = 66 666 666min 40s = 1 111 111h 6min 40s = 46 296 days 7h 6min 40s
Now 2150 to 2097 is 53 years (19 345 days). Every leap year now has an extra day.
So every 4 years there are 1461days. 18 × 1461 = 26 298days (Another 72 years).
This leaves us 653 days (We are now at 2025). As 2024 is a leap year we arrive at 18 May 2023.

2. ADAM'S ANTICS



ΔCBE and ΔACD are congruent right angled Δs
Thus angle BCA = 90°
Thus BA = √2m

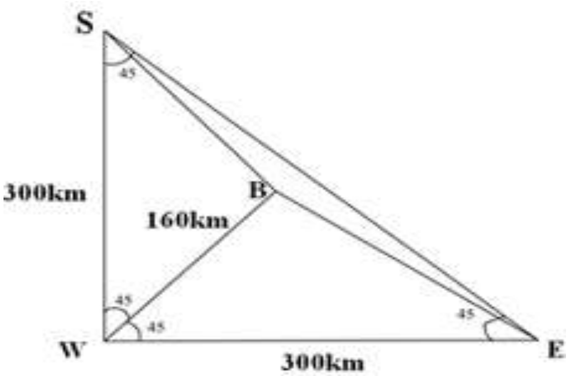
$$\text{Arc BA} = \frac{90}{360} \times 2\pi \times 1 = \frac{\pi}{2}\text{m}$$

Thus shortest distance for round trip is

$$= \sqrt{2} + \frac{\pi}{2}\text{ m} = 2.985\text{m (to 3d.p.)}$$

To avoid going uphill Adam's shortest route would be half the circumference of the small circle with BA as diameter. BA = √2m Semicircle BA = $\frac{\sqrt{2}}{2}\pi$m Thus shortest distance for round trip is $\sqrt{2}+\frac{\sqrt{2}}{2}\pi$m = 3.636m

3. SHARP RETURN



ΔSWE is isosceles so SW=300km

In ΔBWS $BS^2 = 300^2 + 160^2 - 2\times300\times160\cos45°$
$= 47\ 717.749$

Thus BS = 218.444km (to 3d.p.)

So the average speed is $\frac{218.444}{15}$km/min

Time to travel S→W $=\frac{300\times15}{218.444} = 20.6$min

Thus total time taken is 2×20.6+2×15 +75=146.2min
Hence he arrives back at 16.26 (and 12s) so he has just under 13 minutes to arrive at the squash court.

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4. BACKWARDS………..

Let a,b ∈ N and 1≤a≤9 and 0≤b≤9
Any 4 digit palindromic number will be of the form abba
Now abba = 1000a + 100b + 10b + a = 1001a + 110b = 11(91a + 10b)
Thus every 4 digit palindromic number has a factor of 11
Also abba = 1001a + 110b = 13×77a + 13×8b + 6b = 13(77a + 8b) +6b
Thus 4 digit palindromic numbers only have a factor of 13 when 6b = 0 (i.e. b = 0)
This would be 2002 and 1001.
Hence they can visit 9 years; 1991, 1881, 1771, 1661, 1551, 1441, 1331, 1221, 1111.

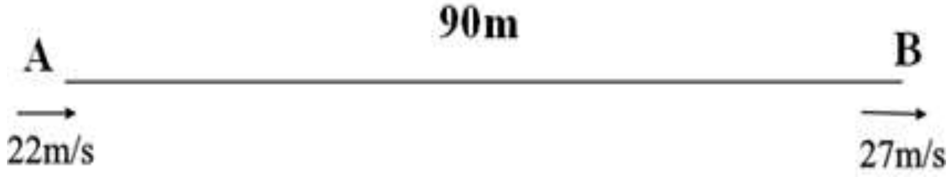
……..AND FORWARDS

Now abba = 1001a + 110b = 999a + 108b + 2a +2b = 3(333a + 36b) + 2(a + b)
Thus 4 digit palindromic numbers have a factor of 3 when a + b has a factor of 3.
It can be shown that there are 30 ordered pairs hence there are 30 visits.
Now aba = 101a + 10b = 3×33a + 3×3b + 2a + b = 3(33a + 3b) + 2a + b
Thus 3 digit palindromic numbers have a factor of 3 when 2a + b has a factor of 3.
It can be shown that there also 30 ordered pairs of this type.
The two teams make the same number of visits.

5. ISLAND HOPPING

In any triangle the longest side is always opposite the largest angle. If an island is at the centre of a regular pentagon then the angle at the centre is 72° and the distance between the outer islands can be such that this is always greater than the distance to the central one. Thus 5 ferries can arrive at the central island. If the island is at the centre of a hexagon whilst 5 of the angles can each be greater than 60° the sixth one cannot. Thus in this triangle the distance between the outer islands will be less than the distance to the central one.
Hence no island will receive more than 5 ferries in one day.

6. THE FLYING SCOUSER



Using $v^2 = u^2 + 2as$ we have $27^2 = 22^2 + 2 \times a \times 90$ $\therefore a \approx 1.36ms^{-2}$
For the end of the train to pass B the Flying Scouser will have travelled 190m from A.
Using $s = ut + \frac{1}{2}at^2$ we have $190 = 22t + \frac{1}{2} \times 1.36 \times t^2$ $\therefore 0.68t^2 + 22t - 190 = 0$
Solving this quadratic gives t= 7.08seconds.
Thus the Flying Scouser takes approximately 7 seconds to pass through the station.