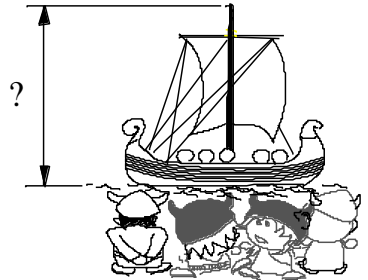


In the final game they began with an odd number of small stones placed between them. Each of them, in turn, removed either one, two or three stones and the winner was the one who ended up with an odd number of stones. If they had started with 15 stones and Eirik chose first, who should have won? What if they had started with 13 stones? What about other starting numbers of stones?

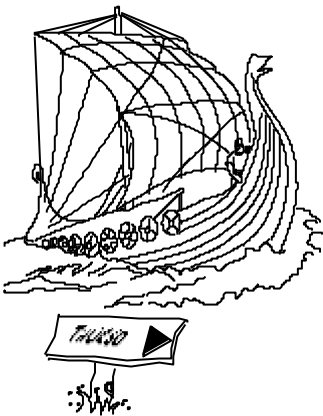
## 5. HOW HIGH CAN YOU GO?

One day four of Knut's sons went down to the river to inspect their father's latest warship. Bjorn wondered if they could work out the height of its mast. They separated and Cnut estimated his distance from the base of the mast was five times its height. Dan said that his distance from the base was two and a half times its height.



Edgar said that this was just what he would have said. Bjorn realised that as each of his three brothers was 50m from the other two he could now work out the height of the mast. What was the height of the mast?

## 6. CROSSING THE SEA



Bjorn's longship launched from a point on Orkney to cross to Thurso, a distance of 25 nautical miles. He sailed at a steady rate of 12 knots for the first 15 nautical miles but then, because of the worsening sea conditions, each succeeding nautical mile took 20% longer to complete than the previous one. How long did it take to complete the first 16 nautical miles? How long did the whole crossing take?

The competition is promoted by Liverpool Mathematical Society (LMS)

[www.maths.liv.ac.uk/lms.html](http://www.maths.liv.ac.uk/lms.html)

The Liverpool Mathematical Society incorporates the Liverpool Branch of the Mathematical Association.  
The MA is a Registered Charity (No. 313281).

# Open Challenge '14 For Year 13 or below

Illustrations by Peter H Ackerley

## Rules

- 1) It should be attempted at home during February half term.
- 2) Your entry must be your own work.
- 3) For individual entries you should attempt any four questions.  
For team entries (two or more students) you should attempt all six questions.
- 4) Entries without any working out at all or written on this sheet will not be marked.
- 5) It is possible to win a prize even if you have not completed all of the questions, so hand in your entry even if it is not quite finished.
- 6) You must write **your name(s), date(s) of birth and school in neat, legible writing on the front sheet.**
- 7) Pupils under 15 years of age should only attempt this in exceptional circumstances.

Either you or your maths teacher needs to return your entry by 7 March to this address:

### Open Challenge '14 Entries,

Mrs A. Carter,  
Danes Court,  
Mudhouse Lane,  
Burton,  
Neston.  
CH64 5TS.

All of the prizes and certificates will be awarded at an evening of mathematical recreation at the University of Liverpool on 7 May. Solutions will be posted on

[www.maths.liv.ac.uk/lms.html](http://www.maths.liv.ac.uk/lms.html) shortly afterwards. We hope that you enjoy the questions.

## 1. EIRIK'S SHIELD

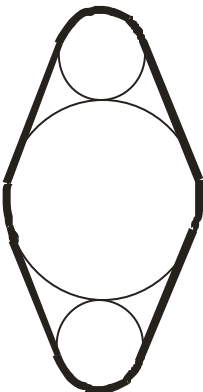
Eirik was not the strongest of all his tribe but he was clever and he was Olaf's champion. He had to fight Arfast, the mightiest Viking in the land, who had challenged Olaf. Eirik had the legendary shield, Grimold, for protection. It had been crafted in the dim and distant past and was made up of a number of different pieces of wood all of different sizes. Grimold had a magical quality as the areas of the individual pieces formed a consecutive sequence of prime numbers with a total area of less than 1000 units.



The fight began and Arfast's first stroke was so mighty that pieces representing half the area of Grimold fell to the ground. A second stroke caused half the remaining area to fall. A third stroke similarly caused half the remaining area to fall. A fourth blow caused three quarters of the remaining area to fall. Now Eirik only had a single piece of his shield remaining but Arfast was tiring so Eirik rushed in and dispatched him with a single blow.

What was the size of the last piece of shield and what were the areas that fell to the ground with each blow?

## 2. A WORK OF ART

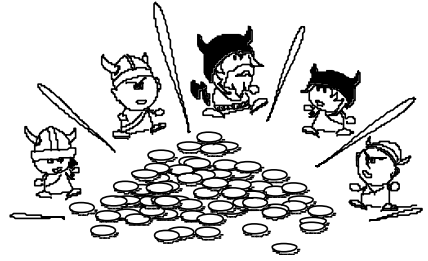


Olaf asked for a special trophy to be made for Eirik. It was to be made from three discs of silver. The central disc was of radius 9cm and the other two were of radius 4cm. The three discs were to be surrounded by a gold band of width 1cm.

What was the length of the gold band?

### 3. KNUTSONS HOARD

After feasting Knut told Olaf of the time when his five sons Alfrain, Bjorn, Cnut, Dan and Edgar had seized a hoard of gold coins and as usual fought each other over the spoils. Alfrain took hold of two thirds of the coins but Bjorn grabbed three eighths of these and Cnut managed to seize three tenths also.

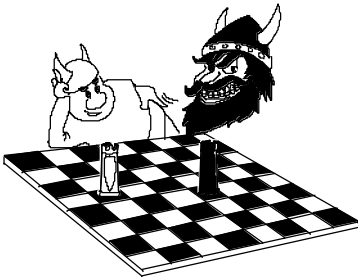


Young Dan dashed in and snatched all that Alfrain had left except one seventh which Edgar took. Now Alfrain and Cnut set upon Bjorn, who dropped one half of what he had which were equally picked up by Dan and Edgar. Next Bjorn sprang on Cnut who upset all his collection on the floor. Of these Alfrain got just one quarter, Bjorn gathered one third, Dan got two sevenths while Cnut and Edgar divided equally what was left. Suddenly Dan struck out in two directions at once upsetting three quarters of what Bjorn and Alfrain had last acquired. These two recovered five eighths of it in equal shares but the other three each took one third of the rest.

On this Knut had entered, a truce was called and the five sons divided the remainder of the coins equally among them.

What was the smallest number of coins there could have been in the hoard and what proportion did each son obtain?

### 4. BEST OF THREE



Eirik's son, Bram, was clever like his father and the two of them often played games. Bram challenged his father to a game of Fox and Geese which he won. Eirik then challenged him to a game using two rooks on an 8x8 chessboard. These rooks, one black and one white, can be placed anywhere on the chessboard. The object of the game was to capture your opponent's piece. You take it in turns to move but you cannot cross your opponent's 'line of fire' either horizontally or vertically. How did Eirik make sure he would win?