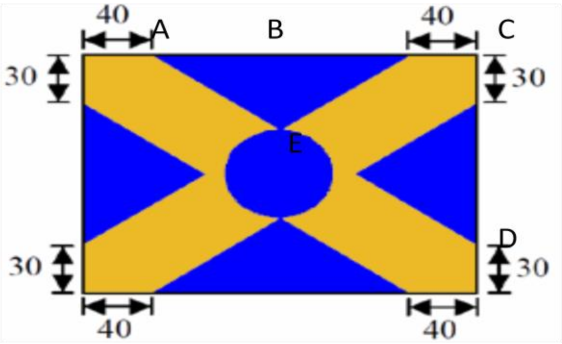
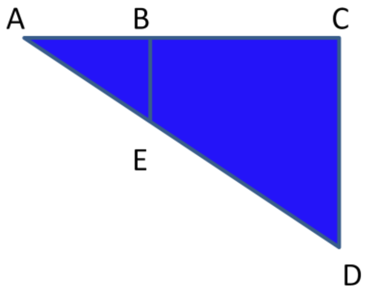
OPEN CHALLENGE '12
SOLUTIONS

1. Wave the Flag





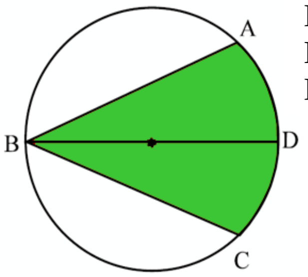
△ABE is similar to △ACD
(∠EAB=∠DAC &
∠ABE=∠ACD)

$$\frac{BE}{CD}=\frac{BA}{CA}$$
$$\frac{BE}{120}=\frac{80}{200}$$

⇒ BE= 48cm
as BE + r = 75
radius of circle is 27cm

Again using similar triangles it can be shown that the height of the side triangles is 75cm
To calculate the area of the cross, the areas of the four triangles are subtracted from the area of the rectangle.
Thus Area of cross $= 240\times150 - (2\times0.5\times90\times75 + 2\times0.5\times160\times48)$
$= 21\,570\text{cm}^2$

2. Hammer it out

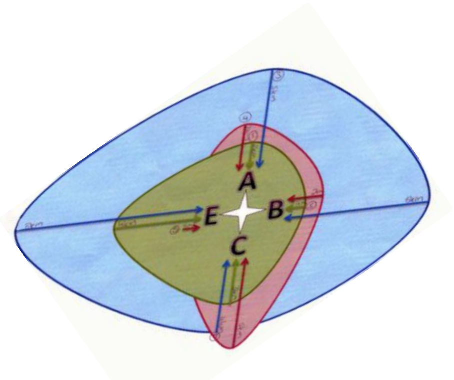


Let ∠ABC = θ (in radians)
Let O be the centre of the circle
Let r be the radius of the circle

Area of △AOB + Area of △BOC $= 2\times0.5\times r^2\times\sin(\pi-\theta)$
$= r^2\sin\theta$
Area of Sector AOC $= 0.5r^2\times2\theta$
$= r^2\theta$

Thus shaded area is $r^2\sin\theta + r^2\theta = 0.375\,\pi\, r^2$
Thus $\sin\theta + \theta = 0.375\,\pi$
This can be solved graphically or by trial and improvement
This gives a value of θ=0.6074 radians
θ=34.80° (to 2d.p.)
The official size of the angle is 34.92°
Thus percentage error $= \frac{34.92-34.80}{34.92}\times100$
= 0.34% (to 2d.p.)

3. Getting there



There are a number of ways of solving this which are based on minimizing the distances travelled.
The girls from Manchester High School came up with their 'Sphere of Travel'
which illustrates the idea in a neat fashion.

	Archery	Badminton	Cycling	Equestrian
The Royal	4			5
The Sovereign	1	5		
The Tower	3		7	

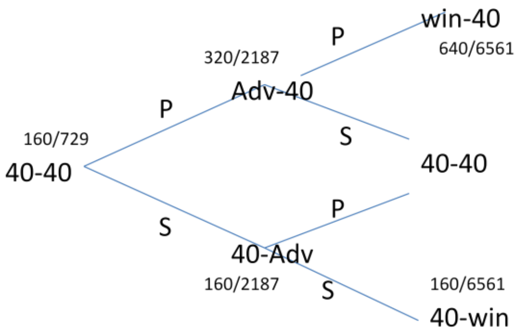
There are a number of solutions similar to this one which all give the total distance travelled as 67km.

4. Anyone for tennis?

In theory a game of tennis may never end as the score can always return to 'deuce'.
The probability of Paris or Sonya winning the game is given by:

P(win/0) + P(win/15) + P(win/30) + P(win/40)

The probability of Paris winning the game = 16/81 + 64/243 + 160/729 + S,
where S is the sum of an infinite geometric progression



$$S_\infty = \frac{a}{1-r}$$

For Paris we have $S_\infty = \frac{640/6561}{1-4/9} = 128/729$

Thus for a probability of a Paris win we have
P(Paris) =16/81 + 64/243 + 160/729 + 128/729
= 208/243 (0.856 to 3d.p.)
Probability of a Sonya win is 35/243 (0.144 to 3d.p.)

5. Dive in

If Bianca's first statement is true then she was second (not third);
and if her second statement is true then Aroha is third (not Bianca).
Thus Bianca's statements prove she was not third.
Diane's statement that Bianca was third is false so Aroha must be fourth.
This then implies that Bianca must be second.
Thus Aroha's correct statement gives Diane as last.
1st Ekaterina, 2nd Bianca, 3rd Cong, 4th Aroha and 5th Diane.

6. And finally

In a tennis match the possible winning scores in the first four sets are; 6-0, 6-1, 6-2, 6-3, 6-4, 7-5 or 7-6.
Since the minimum number of games is 6 and the maximum number is 13 the only possible scores for the first two sets are 6-0 and 7-5.
Let the number of games played in set 4 be x then the number played in set 5 is 2x
Thus we have $6 \le 48 - 3x \le 13$ thus $35 \le 3x \le 42$
As x has to be a whole number less than 14 the only possible values of x are 12 or 13.
It can be shown that with x equal to 12 the number of games won by each player must be even (not possible) and thus x must be 13.
This gives set 4 as 7-6 and set 5 as 14-12.
Thus Menem wins the final 0-6, 7-5, 6-3, 6-7 and 14-12.